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Direction-consistent tangent vectors for generating interpolation curves

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ABSTRACT

In this paper, monotonicity-preserving interpolation is generalized to direction-consistent interpolation. The conditions for constructing direction-consistent tangent vectors are given. The conditions on the tangent vectors are also obtained such that the piecewise cubic Hermite interpolation curves are tangent direction-consistent with the direction of data polygon. Based on geometric insights, the balanced tangent vectors are presented and proved to be direction-consistent tangent vectors. With the balanced tangent vectors, the generated cubic Hermite interpolation curves are tangent direction-consistent, and the generated quintic Hermite interpolation curves are also tangent direction-consistent provided that we use the accumulative chord length parametrization. Some graphic examples are given to show that the generated interpolation curves preserve satisfactorily the shape of the given data control polygon.

1. Introduction

On the problem of tangential expression of interpolation curves, a remarkable result was Bessel tangents, see [1] and [2]. There are some other methods to compute the derivatives of the cubic Hermite interpolant. For smooth fitting of data, a method was proposed by Akima in [3]. In [4], the derivatives were determined such that the cubic Hermite interpolant was the least squares approximation of the piecewise linear interpolant. The least squares solution of the derived function of the cubic Hermite interpolant was discussed in [5]. The optimization at the endpoints of Akima's method was proposed in [6]. The shape-preserving C^1 splines were studied by [7]. In [8], control parameters (tension, bias, continuity) were considered. The monotonic splines with minimal curvature-type strain energy of the curve were discussed in [9].

There are many methods for constructing shape-preserving interpolation. Some methods are appropriate for monotonicity-preserving, see [10–13]. Rational interpolant can produce satisfied shape-preserving interpolation. In the paper [14], the given interpolant was convexity-preserving and C^2 continuous without solving a global system of equations. Different rational interpolation methods with shape parameters were presented in [15] and [16]. Usually, a method needs more than two consecutive monotone or convex/concave data for constructing shape-preserving interpolant. In [17], without shape parameter, the cubic spline method for convexity-preserving approximation was proposed.

In the application to computer-aided design, some methods for constructing shape-preserving interpolation have been presented, see [18–26]. The shape-preserving property is achieved by adjusting tension parameters, see [18,19,21,22]. For a measure of the oscillations of spline function, the deviations from the data polygon were studied by [27]. In [28], optimized geometric Hermite curves with minimum strain energy were constructed. To assess curve quality, some methods based on curvature or energy analysis were discussed, see [29–33].

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Although there are a lot of researches on the shape-preserving curve representation, the simple cubic polynomial interpolation methods with direction-consistent tangent vectors have not been discussed. The motivation of this paper is to present an interpolation method by using direction-consistent tangent vectors. The method can be viewed as a new alternative to the shape-preserving interpolation methods and is generalized from one-dimensional interpolation to parametric curve interpolation.

The rest of the paper is organized in the following way. In Section 2, the direction-consistent tangent vectors are presented. For the given data, the conditions for constructing direction-consistent tangent vectors are given. Based on geometric insights, the balanced tangent vectors are presented. In Section 3, the conditions on tangent vectors are obtained such that the piecewise cubic Hermite interpolation curves are tangent direction-consistent. By using the balanced tangent vectors, the generated cubic Hermite interpolation curves are proved to be tangent direction-consistent with the direction of the data polygon. In Section 4, with accumulative chord length parametrization and the balanced tangent vectors, the generated quintic Hermite interpolation curves are proved to be tangent direction-consistent. Some graphic examples are given to show that the generated interpolation curves preserve satisfactorily the shape of the given data control polygon in Sections 3 and 4 respectively. The conclusions are given in Section 5.

2. Direction-consistent tangent vectors

Monotonicity-preserving interpolation is an interesting topic which many research results have been achieved in the recent forty years. Now we generalize the problem of monotonicity-preserving to the case on vector-valued data. We will use the inner product notation $\langle X, Y\rangle = X^TY$, where X and Y are vectors. For Euclidean norm, an inner product obeys $\langle X, X\rangle = \|X\|^2$, For two-dimensional vectors, an inner product obeys $\langle X, Y\rangle = \|X\|\|Y\|\cos\theta$, where θ is the included angle between X and Y.

Given data points $P_1, P_2, \ldots, P_n$ with corresponding parametric values $u_1, u_2, \ldots, u_n$, let $h_i = u_{i+1} - u_i$, $\Delta P_i = (P_{i+1} - P_i)/h_i$, $i = 1, 2, \ldots, n-1$, it is known that ΔP_i represents the tangent direction of the piecewise linear interpolant $(1-t)P_i + tP_{i+1}$ for $t = (u-u_i)/h_i$, $u \in [u_i, u_{i+1}]$. The piecewise linear interpolation curves represent the shape of the given data. Therefore we expect that the tangent direction of an interpolating curve approaches the direction of the vector ΔP_i for $u \in [u_i, u_{i+1}]$.

Definition 1. Let $C(u)$ be an interpolating curve passing through the data points P_i at $u = u_i$, $i = 1, 2, \ldots, n$, then the curve $C(u)$ is called tangent direction-consistent on $u \in [u_i, u_{i+1}]$ if $\langle \Delta P_i, C'(u)\rangle \geq 0$.

Here $C(u)$ is a general interpolating curve. Cubic Hermite interpolation and quintic Hermite interpolation will be discussed in the subsequent sections.

Let D_i be the tangent vectors of $C(u)$ at $u = u_i$, $i = 1, 2, \ldots, n$, then the conditions $\langle \Delta P_i, D_i\rangle \geq 0$ are necessary from Definition 1. For the given interior data points, there are two directions ΔP_{i-1} and ΔP_i at the point P_i. Naturally, we want that the direction of a tangent vector at P_i approaches the directions of ΔP_{i-1} and ΔP_i.

Definition 2. For $i = 1, 2, \ldots, n$, we call vectors D_i as direction-consistent tangent vectors at P_i if

$$\langle D_1, \Delta P_1\rangle \geq 0, \quad \langle D_i, \Delta P_{i-1}\rangle \geq 0, \quad \langle D_i, \Delta P_i\rangle \geq 0, \quad \langle D_n, \Delta P_{n-1}\rangle \geq 0, \tag{1}$$

where $i = 2, 3, \ldots, n-1$.

The conditions of Definition 2 are considered for the case on open data. For the closed data with P_n consecutive by P_1, all points are interior data. We do not need to consider the conditions for the endpoints, and can extend the conditions for interior data to $i = 1, 2, \ldots, n$ by setting $P_0 = P_n$, $P_{n+1} = P_1$, $u_0 < u_1$ and $u_{n+1} > u_n$. In the following, we discuss the case on open data and extend the processing on interior data for closed data.

From Definition 2, the direction-consistent tangent vector is the vector that its direction is consistent with the directions of the adjacent two edges of the data polygon on the meaning of (1). When $P_i \in \mathbb{R}$, from (1) we specify that the direction-consistent tangent vector $D_i \geq 0$ for monotone increasing data P_{i-1}, P_i, P_{i+1} and $D_i \leq 0$ for monotone decreasing data P_{i-1}, P_i, P_{i+1}. In the previous research on monotonicity-preserving interpolation, it was focused on constructing monotonicity-preserving interpolants while $D_i\Delta P_i \geq 0$ was regarded as natural facts for monotone data.

Some existing tangents may not obey the condition (1). Among the local tangents, the Bessel tangents $\bar{D}_i$ are important tangents and have been used widely. The Bessel tangents $\bar{D}_i$ are chosen as the derivatives at u_i of the parabolas Q_i that agree with P_{i-1}, P_i, P_{i+1} for $i = 2, 3, \ldots, n-1$. The endpoints are treated as in the way: $\bar{D}_1$ and $\bar{D}_n$ are the derivatives at u_1 and u_n of the parabolas Q_2 and Q_{n-1} respectively. A short calculation gives

$$\bar{D}_1 = \frac{2h_1 + h_2}{h_1 + h_2}\Delta P_1 - \frac{h_1}{h_1 + h_2}\Delta P_2,$$
$$\bar{D}_i = \frac{h_i}{h_{i-1} + h_i}\Delta P_{i-1} + \frac{h_{i-1}}{h_{i-1} + h_i}\Delta P_i, \quad i = 2, 3, \ldots, n-1,$$
$$\bar{D}_n = \frac{h_{n-2} + 2h_{n-1}}{h_{n-2} + h_{n-1}}\Delta P_{n-1} - \frac{h_{n-1}}{h_{n-2} + h_{n-1}}\Delta P_{n-2}.$$