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Numerical simulation of a compressible two-layer model: A first attempt with an implicit–explicit splitting scheme

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ABSTRACT

This paper is devoted to the numerical simulation of the compressible two-layer model developed in Demay and Hérard (2017). The latter is a hyperbolic two-fluid two-pressure model dedicated to gas–liquid flows in pipes, especially stratified air–water flows. Using explicit schemes, one obtains a CFL condition based on the celerity of (fast) acoustic waves which typically brings large numerical diffusivity for the (slow) material waves and small time steps. In order to overcome these drawbacks, the proposed scheme involves an operator splitting and an implicit–explicit time discretization. Thus, the full system is split into two hyperbolic sub-systems. The first one deals with the transport equation on the liquid height using an explicit scheme and upwind fluxes. The second one deals with the averaged mass and momentum conservation equations of both phases using an implicit scheme which handles the propagation of acoustic waves. At last, the positivity of heights and densities is ensured under a CFL condition which involves material velocities. Numerical experiments are performed using acoustic as well as material time steps. Adding the Rusanov scheme for comparison, the best accuracy is obtained with the proposed scheme used with acoustic time steps. Focusing on material waves of the convective system, the efficiency of the latter is improved when using material steps. However, considering the whole system with relaxation source terms, an efficient approximation of slow dynamics, typically a gravity driven flow, is still challenging.

1. Introduction

In this work, we focus on the compressible two-layer model developed in [1]. The latter deals with transient gas–liquid flows in pipes, especially stratified air–water flows which occur in several industrial areas such as nuclear power plants, petroleum industries or sewage pipelines. It is a five-equation system which results from a depth averaging of the isentropic Euler set of equations for each phase where the classical hydrostatic assumption is applied to the liquid. This system is composed of a transport equation on the liquid height in addition to averaged mass and averaged momentum conservation equations for both phases. The derivation process presents similarities with the work exposed in [2]. Thus, the resulting model is a two-fluid two-pressure model and displays the same structure as an isentropic Baer–Nunziato model which provides a statistical description of two-phase flows, especially granular flows or bubbly flows (see for instance [3,4]). In this

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context, interesting mathematical properties are obtained such as hyperbolicity, entropy inequality, explicit eigenstructure as well as Riemann invariants and uniqueness of jump conditions. Note that the numerical discretization of this compressible two-layer model has not been considered yet in the literature such that the work presented herein is a first attempt.

From a numerical point of view, the compressible two-layer model, as the isentropic Baer–Nunziato model, is complex to deal with for several reasons. The first difficulty arises from the large size of the system which makes the Riemann problem difficult to solve regarding the convective part and Godunov-type methods. The second difficulty is linked to the presence of non-conservative products in the governing equations such that the model does not admit a full conservative form. However, the non-conservative products vanish and the system reduces to two decoupled isentropic Euler-type systems on both sides of a linearly degenerate field which is parameterized using the corresponding Riemann invariants. The third difficulty results from the non-linearity in pressure laws which renders even more difficult the derivation of Riemann solvers. When dealing with the full system, one also has to account for relaxation phenomena, in particular pressure relaxation and velocity relaxation given by the source terms, which bring numerical issues regarding the involved time scales.

Despite the mentioned difficulties, some successful solvers are proposed in the literature focusing on the convective part of the Baer–Nunziato system. They are mainly time-explicit Godunov-type methods such as Roe-like scheme, HLL or HLLC scheme and relaxation scheme, see [5–10] among others. For stability reasons, such methods have to comply with the usual Courant–Friedrichs–Lewy (CFL) condition on the time step which involves the celerity of (fast) acoustic waves and can be very restrictive. In our framework of two-layer pipe flows, even if we are interested in the accurate description of fast waves when the pipe is full of water (in water hammer situation for instance), we are also interested in the dynamics of slow waves associated to material velocities. Thus, an additional difficulty relies in the mix of two types of waves, namely the (fast) acoustic and the (slow) material waves. A possible way to tackle this issue is to use a fractional step method or equivalently an operator splitting. It consists in a multi-step algorithm where each step deals with a system containing exclusively acoustic or material waves. This approach is developed in [11,12] for the Euler model and in [13] for the isentropic Baer–Nunziato model, among others. Note also that some similarities may be found with the so-called flux splitting approach used in [14] for the Euler model and recently in [15] for the Baer–Nunziato model. However, the above-mentioned references are explicit in time and the CFL condition on the time step still relies on the celerity of fast waves. In order to obtain a less restrictive CFL condition, an implicit–explicit scheme may be used where the fast waves are treated implicitly and the slow waves explicitly to preserve accuracy. Combining the splitting approach and the implicit–explicit treatment, one obtains a CFL condition based on material velocities and consequently a large time-step scheme. This was initially proposed in the context of the Euler model, see [16–18], and an extension to the Baer–Nunziato model was proposed in [19]. Particularly, the latter reference uses a Lagrange-Projection approach that consists in approximating the gas dynamics equation using the Lagrange coordinates and then remapping the solution onto an Eulerian mesh. Note that implicit–explicit strategies are also used to derive all speed or all Mach schemes with asymptotic preserving properties regarding the compressible Euler model and its incompressible limit, see [20–22,17]. Thus, one can obtain accurate schemes in the low Mach regime with large time steps. Nonetheless, such low Mach properties are still difficult to acquire for two-fluid two-pressure models as the limit model is not clearly defined.

The work presented herein provides numerical results regarding the compressible two-layer model and the related challenges exposed above. Thus, in addition to consider a classical explicit Rusanov scheme known for its robustness, i.e. a first-order finite volume scheme with Rusanov fluxes [23,24], we propose a large time-step implicit–explicit scheme relying on an operator-splitting approach. The five-equation system is split into two hyperbolic sub-systems. The first one deals with the transport equation on the liquid height using an explicit scheme and upwind fluxes. The second one deals with the averaged mass and momentum conservation equations using an implicit scheme which handles the propagation of acoustic waves. At the end, the positivity of heights and densities is ensured under a CFL condition which involves material velocities. Numerical experiments with grid convergence studies are performed with both schemes using analytical solutions for the convective part of the system. The source terms are then handled accounting for the interactions between the convective dynamics and relaxation phenomena. The dambreak test case is first considered where the numerical solutions are compared with a reference solution given by the incompressible one-layer shallow-water system. Secondly, one considers a so-called mixed flow test case which involves a transition to the pressurized regime (pipe full of water) through a pipe filling.

The chapter is organized as follows. The governing equations of the model under consideration are recalled in Section 2 as well as its main mathematical properties. Focusing on the convective part of the system, the splitting approach and the associated implicit–explicit scheme are presented in Section 3. Numerical experiments are then performed in Section 4 building analytical solutions thanks to the available jump conditions and Riemann invariants. In the last part, the full model with the source terms is handled and tested against the dambreak problem and a mixed-flow configuration.

2. The compressible two-layer model

The considered model deals with stratified gas–liquid flows in pipes, see Fig. 1 for a typical configuration. It results from a depth-averaging of the isentropic Euler set of equations for each phase where the classical hydrostatic assumption is made for the liquid, see [1] for details. The governing equations of the model and its main mathematical properties are exposed below.