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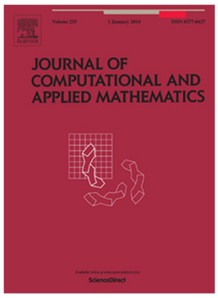

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GENERALIZED CONFORMABLE FRACTIONAL OPERATORS

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ABSTRACT. In [1] T. Abdeljawad has put an open problem, which is stated as: "Is it hard to fractionalize the conformable fractional calculus, either by iterating the conformable fractional derivative (Grunwald-Letnikov approach) or by iterating the conformable fractional integral of order $0 < \alpha \leq 1$ (Riemann approach)?. Notice that when $\alpha = 0$ we obtain Hadamard type fractional integrals".
In this article we claim that yes it is possible to iterate the conformable fractional integral of order $0 < \alpha \leq 1$ (Riemann approach), such that when $\alpha = 0$ we obtain Hadamard fractional integrals. First of all we prove Cauchy integral formula for repeated conformable fractional integral and proceed to define new generalized conformable fractional integral and derivative operators (left and right sided). We also prove some basic properties which are satisfied by these operators. These operators (integral and derivative) are the generalizations of Katugampola operators, Riemann-Liouville fractional operators, Hadamard fractional operators. We apply our results to a simple function. Also we consider a nonlinear fractional differential equation using this new formulation. We show that this equation is equivalent to a Volterra integral equation and demonstrate the existence and uniqueness of solution to the nonlinear problem. At the end, we give conclusion and point out an open problem.

1. Introduction and Preliminaries

The idea of fractional calculus has impelled a host of researchers towards it for a last few decades. Work has been carried out on it on a large scale and everyone has awakened its various aspects. The contributions of Euler, Laplace, Fourier, Abel, Liouville, Riemann, Grunwald, Letnikov, Hadamard and in the present century, Weyl, Riesz, Marchaud, Kober and Caputo are remarkable in this field [1–10]. Most of these researchers initially introduced fractional integrals, on the basis of which the associated fractional derivative and other related results were produced. Some of the most explored and commonly used definitions of fractional integrals are given below.
The right-sided Riemann-Liouville fractional integral operator of order $\beta > 0$ is given by [20]:

$$J_{p^+}^{\beta}\phi(r) = \frac{1}{\Gamma(\beta)} \int_p^r (r-w)^{\beta-1}\phi(w)dw, \quad \text{with} \quad r > p, \tag{1}$$

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