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Signal Recovery under Cumulative Coherence

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Abstract This paper considers signal recovery in the framework of cumulative coherence. First, we show that the Lasso estimator and the Dantzig selector exhibit similar behavior under the cumulative coherence. Then we estimate the approximation equivalence between the Lasso and the Dantzig selector by calculating prediction loss difference under the condition of cumulative coherence. And we also prove that the cumulative coherence implies the restricted eigenvalue condition. Last, we illustrate the advantages of cumulative coherence condition for three class matrices, in terms of the recovery performance of sparse signals via extensive numerical experiments.

Keywords Cumulative coherence · Dantzig selector · Lasso · Oracle inequality · Restricted eigenvalue condition · Closeness of prediction loss

Mathematics Subject Classification 62G05 · 94A12

1 Introduction

Compressed sensing predicts that sparse signals can be reconstructed from what was previously believed to be incomplete information. Since Candès, Romberg and Tao's seminal works [9, 10] and Donoho's ground-breaking work [18], this new field has triggered a large research in mathematics, engineering and medical image. In such contexts, we often require to recover an unknown signal $x \in \mathbb{R}^n$ from an underdetermined system of linear equations

$$b = Ax + z, \tag{1.1}$$

where $b \in \mathbb{R}^m$ are available measurements, the matrix $A \in \mathbb{R}^{m\times n}$ $(m < n)$ models the linear measurement process and $z \in \mathbb{R}^m$ is a vector of measurement errors.

For the reconstruction of x, the most intuitive approach is to find the sparsest signal in the feasible set of possible solutions, which leads to an ℓ_0-minimization problem as follows

$$\min_{x\in\mathbb{R}^n} \|x\|_0 \ \text{ subject to } \ b - Ax \in \mathcal{B},$$

where $\|x\|_0$ denotes the ℓ_0 norm of x, i.e., the number of nonzero coordinates, and $\mathcal{B}$ is a bounded set determined by the error structure. However, such method is NP-hard and

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