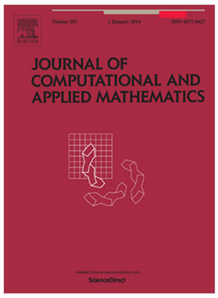

Off-diagonal low-rank preconditioner for difficult PageRank problems

Zhao-Li Shen, Ting-Zhu Huang, Bruno Carpentieri, Chun Wen, Xian-Ming Gu, Xue-Yuan Tan

PII: S0377-0427(18)30435-7
DOI: https://doi.org/10.1016/j.cam.2018.07.015
Reference: CAM 11801

To appear in: *Journal of Computational and Applied Mathematics*

Received date : 26 February 2017
Revised date : 16 December 2017

Off-diagonal low-rank preconditioner for difficult PageRank problems

Zhao-Li Shen[a,b], Ting-Zhu Huang[a,*], Bruno Carpentieri[c], Chun Wen[a], Xian-Ming Gu[b,d], Xue-Yuan Tan[e]

[a] *School of Mathematical Sciences, University of Electronic Science and Technology of China, Chengdu, Sichuan 611731, P.R. China*
[b] *Johann Bernoulli Institute for Mathematics and Computer Science, Faculty of Science and Engineering, University of Groningen, 9700 AK Groningen, The Netherlands*
[c] *School of Science and Technology, Faculty of Computer Science, Free University of Bozen-Bolzano, 39100 Bolzano, Italy*
[d] *School of Economic Mathematics, Southwestern University of Finance and Economics, Chengdu, Sichuan 611130, P.R. China*
[e] *Jiangsu Key Laboratory for NSLSCS, School of Mathematical Science, Nanjing Normal University, Nanjing 210046, P.R. China*

Abstract

PageRank problem is the cornerstone of Google search engine and is usually stated as solving a huge linear system. Moreover, when the damping factor approaches 1, the spectrum properties of this system deteriorate rapidly and this system becomes difficult to solve. In this paper, we demonstrate that the coefficient matrix of this system can be transferred into a block form by partitioning its rows into special sets. In particular, the off-diagonal part of the block coefficient matrix can be compressed by a simple low-rank factorization, which can be beneficial for solving the PageRank problem. Hence, a matrix partition method is proposed to discover the special sets of rows for supporting the low-rank factorization. Then a preconditioner based on the low-rank factorization is proposed for solving difficult PageRank problems. Numerical experiments are presented to support the discussions and to illustrate the effectiveness of the proposed methods.

Keywords: PageRank, Off-diagonal, Low-rank factorization, Matrix partition, Preconditioner

1. Introduction

With the rapid development of the Internet, Web search engines become very popular for information retrieval [1]. Because a web search engine can usually find an immense set of Web pages matching the search query, it is necessary to rank higher the most important pages. For this purpose, Google has presented the PageRank model (also called PageRank problem) that employs the link structure of Web pages to quantify the importance of each one.

The detailed mathematical background of the PageRank problem can be found in [2], here we give a brief review. The link structure of the related Web pages is represented by a directed graph named the Web link graph. Denote its adjacency matrix by $G \in \mathbb{N}^{n\times n}$ where n is the number of nodes (pages), then $G(i,j)$ is nonzero (being 1) only when page j has a link pointing to page i. Then the transition matrix $P \in \mathbb{R}^{n\times n}$ with respect to the Web link graph is defined as

$$P(i,j) = \begin{cases} \frac{1}{\sum_{k=1}^{n} G(k,j)}, & \text{if } G(i,j) = 1, \\ 0, & \text{otherwise.} \end{cases} \tag{1}$$

*Corresponding author
Email addresses: szlxiaoyao@163.com (Zhao-Li Shen), tingzhuhuang@126.com (Ting-Zhu Huang)