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A single-step iteration method for non-Hermitian positive definite linear systems *

Xiang Wang[†] Xiao-Yong Xiao[‡] Qing-Qing Zheng[§]

Abstract. An efficient single-step iteration method is presented for solving the large sparse non-Hermitian positive definite linear systems. We theoretically prove that this method converges to the unique solution of the system of linear equations under suitable restrictions. Moreover, we derive an upper bound for the spectral radius of the new iteration matrix. Furthermore, we consider acceleration of the new iteration by Krylov subspace methods and some special properties of the new preconditioned matrix are proposed. Numerical experiments on a few model problems are presented to further examine the effectiveness of our new method.

Key words. Non-Hermitian matrix; convergence theory; matrix splitting; preconditioner; numerical experiment

AMS subject classifications. 65F10, 65F50

1 Introduction

In this paper, we consider the following linear system

$$Ax = b, \tag{1}$$

where $A \in C^{n\times n}$ is a non-Hermitian positive definite matrix (that is, the Hermitian part of A is positive definite), and $x \in C^n$ is an unknown vector and $b \in C^n$ is a given vector. Many problems in scientific computing result in a system of linear equations as (1). For example, molecular scattering, lattice quantum chromodynamics, quantum chemistry, diffuse optical tomography, FFT-based solution of certain time-dependent PDEs, eddy current problem and so on; see [1, 2, 11, 14, 15, 19–24, 29, 31, 38, 42] and references therein.

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