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ALTERNATING PROJECTION METHOD FOR A CLASS OF TENSOR EQUATIONS*

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Abstract. This paper considers how to solve a class of tensor equations arising from the unified definition of tensor-vector products. Of special interest is the order-3 tensor equation whose solutions are the intersection of a group of quadrics from a geometric point of view. Inspired by the method of alternating projections for set intersection problems, we develop a hybrid alternating projection algorithm for solving order-3 tensor equations. The local linear convergence of the alternating projection method is established under suitable conditions. Some numerical experiments are conducted to evaluate the effect of the proposed algorithm.

Key words. tensor-vector product, tensor equation, ellipsoid surface, alternating projection method, regularity

AMS subject classifications. 15A69, 65F30, 65F99, 51M04

1. Introduction.

In mathematics, a tensor is usually defined as a multidimensional array [22, 23, 27]. For instance, an order-1 tensor is a vector, an order-2 tensor is a matrix, and tensors of order three or higher are called higher-order tensors. In general, the element (i, j, k) of an order-3 tensor $\mathcal{A}$ is denoted by a_{ijk}, subarrays are formed when a subset of the indices is fixed. As defined in the reference [28], fibers are the higher-order analogue of matrix rows and columns (denoted as $a_{:jk}$, $a_{i:k}$ and $a_{ij:}$ for the order-3 tensor $\mathcal{A}$), slices are two-dimensional sections of a tensor (denoted as $A_{i::}$, $A_{:j:}$ and $A_{::k}$ for the order-3 tensor $\mathcal{A}$).

It is noted that the product of tensor allows the dimensions to be arbitrary, and there are many kinds of tensor products developed for wide applications in chemometrics [22, 23, 27], signal processing [13, 14, 30, 44], computer vision [21, 25, 37, 38, 43], numerical algebra and numerical analysis [7, 12, 19, 24, 45], and other fields as referenced in the literature review [28]. Along with massive applications of the tensor computation, different kinds of definitions on tensor product have been proposed in recent years. One of the most popular definition is the n-mode product of a tensor proposed by Bader and Kolda [3, 28], i.e., multiplying a tensor by a matrix (or a vector) in mode n. The n-mode product of an order-N tensor $\mathcal{A} \in \mathbb{R}^{I_1 \times I_2 \times \cdots \times I_N}$ with a matrix $B \in \mathbb{R}^{J \times I_n}$ is denoted by $\mathcal{A} \times_n B \in \mathbb{R}^{I_1 \times \cdots \times I_{n-1} \times J \times I_{n+1} \times \cdots \times I_N}$, in element-wise,

$$(\mathcal{A} \times_n B)_{i_1 \cdots i_{n-1} j i_{n+1} \cdots i_N} = \sum_{i_n=1}^{I_n} a_{i_1 i_2 \cdots i_N} b_{j i_n}. \tag{1.1}$$

Another useful kind of definition on tensor product is the generalization of matrix product proposed by Shao [41]. For an order-N tensor $\mathcal{A} \in \mathbb{R}^{I \times I \times \cdots \times I}$ and an order-

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