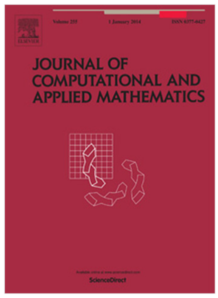
Accepted Manuscript



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PII: S0377-0427(18)30449-7
DOI: https://doi.org/10.1016/j.cam.2018.07.029
Reference: CAM 11815

To appear in: *Journal of Computational and Applied Mathematics*

Received date : 25 August 2017
Revised date : 23 March 2018

Please cite this article as: L. Sun, X. Yan, T. Wei, Identification of time-dependent convection coefficient in a time-fractional diffusion equation, *Journal of Computational and Applied Mathematics* (2018), https://doi.org/10.1016/j.cam.2018.07.029

IDENTIFICATION OF TIME-DEPENDENT CONVECTION COEFFICIENT IN A TIME-FRACTIONAL DIFFUSION EQUATION

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ABSTRACT. In the present paper, we devote our effort to solve a nonlinear inverse problem for identifying a time-dependent convection coefficient in a time-fractional diffusion equation from the measured data at an interior point for one-dimensional case. We prove the existence, uniqueness and regularity of solution for the direct problem by using the fixed point theorem. The stability of inverse convection coefficient problem is obtained based on the regularity of solution for the direct problem and some generalized Gronwall's inequalities. We use a modified optimal perturbation regularization algorithm to solve the inverse convection coefficient problem. Two numerical examples are provided to show the effectiveness of the proposed method.

1. INTRODUCTION

Let $0 < \alpha < 1$, $\Omega = (0, 1)$, $Q_T = \Omega \times (0, T]$. Consider the following initial boundary value problem (IBVP) for a time-fractional diffusion equation with a convection term

$$\begin{cases} \partial_t^\alpha u(x,t) + Au(x,t) - p(t)q(x,t)u_x(x,t) - c(x,t)u(x,t) = 0, & (x,t) \in Q_T, \\ u(0,t) = u(1,t) = 0, & t \in (0,T], \\ u(x,0) = \varphi(x), & x \in \Omega, \end{cases} \tag{1.1}$$

where ∂_t^α denotes the Caputo fractional left-sided derivative defined by

$$\partial_t^\alpha u(x,t) = \frac{1}{\Gamma(1-\alpha)} \int_0^t \frac{\partial u(x,s)}{\partial s} \frac{ds}{(t-s)^\alpha}, \qquad t > 0,$$

in which $\Gamma(\cdot)$ is the Gamma function (see Kilbas et al. [16] and Podlubny [25]) and the differential operator A is defined by

$$Au(x,t) = -\frac{\partial}{\partial x}\left(a(x)\frac{\partial u}{\partial x}(x,t)\right),$$

where the coefficients satisfy

$$a(x) \geq \mu,\ x \in \bar{\Omega},\ a \in C^1(\bar{\Omega})$$

Key words and phrases. Fractional diffusion equation; Inverse problem; Convection coefficient; Modified optimal perturbation algorithm; Stability.