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A Least squares finite element method using Elsasser variables for magnetohydrodynamic equations

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Abstract

There are various different forms of magnetohydrodynamic(MHD) equations and they have been studied for years due to its complicated coupling between variables. This paper proposes to use an equivalently transformed MHD equations with Elsasser variables and the least squares finite element method to find the approximation to them. Introducing new variables by combining fluid velocity and magnetic field yields a Navier-Stokes like system. Then the first-order system least squares method using displacement recasts the transformed MHD equations into a system of first order partial differential equations and the Newton's algorithm linearizes the problem. An L^2-residual functional is defined to minimize and the unique existence of corresponding weak solution is shown. Finally, the convergence of proposed approximation is analyzed and several numerical examples are presented to verify the theory.

Keywords: Least squares finite element approach, The MHD equations, Newton's algorithm

1. Introduction

The MHD equations describe the dynamics of electrically conducting fluids such as solar wind, plasmas, liquid metals, and so on. Due to the various applications for electrically conducting fluids, researchers in various fields are interested in MHD and its applications: magnetic confinement fusion, such as Tokamaks, to confine plasmas and MHD power generation that transforms thermal energy and kinetic energy from electrically conducting fluids into electric power without turbine or rotating machine. The governing equations can be constructed by the Navier-Stokes equations and Maxwell's equations coupled via Ohm's law to describe the motions of electrically conducting fluids. In this paper, we deal with incompressible resistive MHD equations which are second order nonlinear partial equations in three dimensional space.

The finite element method for the stationary, incompressible MHD equations are studied in [12, 13, 21, 24, 29]. In [30], streamline diffusion method is applied in MHD equations to choose arbitrary finite element triples. Several stabilizing techniques for MHD equations were studied in [4, 5, 10, 28] to overcome the difficulties from numerical instabilities which are induced from the small hydrodynamic diffusion. Two-level penalty method for MHD equations was used in [31]. The paper [6] analyzed the uniform stability and optimal error estimates of the Oseen type iterative method and established the error bounds which are explicitly depending on Reynolds numbers and mesh size. Several works for unsteady MHD equations can be found in [22, 23]. A mixed discontinuous Galerkin approach for incompressible MHDs is applied in [12, 15]. An adaptive mesh refinements was utilized to resolve the current sheets effectively in [19, 26]. In [8, 9], least squares approaches were applied to the linear MHD equations and analyzed theoretically. More recently, the least squares methods with nested-iteration-Newton-FOSLS-multigrid-preconditioning in current-vorticity form of MHD equations were analyzed and many numerical results were presented in [1, 2, 3]. A numerical solver for MHD using least squares finite element method was developed in [25]. In applications, least

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