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A NEW AGGREGATION ALGORITHM BASED ON COORDINATES PARTITIONING RECURSIVELY FOR ALGEBRAIC MULTIGRID METHOD

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Abstract. Aggregation based algebraic multigrid is widely used to solve sparse linear systems, due to its potential to achieve asymptotic optimal convergence and cheap cost to setup. In this kind of method, it is vital to construct coarser grids based on aggregation. In this paper, we provide a new aggregation method based on coordinates partitioning recursively. The adjacent graph of the original coefficient matrix is partitioned into sub-graphs and each sub-graph is recursively partitioned until the minimal number of nodes over the sub-graphs on some level is small enough. In this way, a hierarchy of grids can be constructed from top to bottom, which is completely different from the classical schemes. The results from the solution of model partial differential equations with the preconditioned conjugate gradient iteration show that the new algorithm has better performance and is more robust than the widely used classical algorithms in most cases.

Key words. Sparse linear algebraic equations, aggregation based algebraic multigrid, preconditioner, Krylov subspace method, graph partitioning

AMS subject classifications.

1. Introduction. The solution of sparse linear system is the kernel of many scientific and engineering computations. But it is very time-consuming. To solve large sparse linear systems, there are many methods, and the Krylov subspace iterations and the multigrid methods are the most efficient ones.

When Krylov subspace iterations are used, the convergence rate is determined by the distribution of the eigenvalues of the coefficient matrix. The narrower area the eigenvalues are distributed in, the more convergence rate the related method will be. To accelerate the convergence rate, preconditioning techniques are often used, which transfer the linear system to another with better eigenvalue distribution. The preconditioning process is corresponding to an approximate solution to the original system and the more accurate this process is, the less number of iterations there will be. Multigrid is one of such processes and has the potential asymptotic optimal property. It is one of the best combinations to apply it as the preconditioner of Krylov subspace methods.

The efficiency of multigrid methods is determined by the complement of smoothing and correction. The smoothing can reduce the error components with relatively higher frequencies. The rest corresponding to lower frequencies is reduced very slowly and are restricted to a coarser grid to compute the correction [1]. When the smoothing process is given, the efficiency is influenced by two factors, that is, the accuracy of the correction, and the accuracy of the transfer operators.

Algebraic multigrid depends mainly on the algebraic information of the coefficient matrix, and exploits geometry information as little as possible. The aggregation based multigrid is one kind of such methods and is widely focused on, for its cheap cost to setup [2]. In these methods, each coarser grid point is related to several finer grid points, which is called an aggregation. When the linear system is given, the

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