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Stability analysis of MacCormack rapid solver method for evolutionary Stokes-Darcy problem

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Abstract. A numerical method based on the explicit-implicit scheme of MacCormack finite difference scheme was applied to the solution of Parabolized Navier-Stokes equations. Partitioned methods, which solve the coupled problem by successively solving the sub-physics problems, have recently been studied for the full evolutionary groundwater-surface water flow with convergence established over both bounded and long time intervals. A MacCormack rapid solver method based on interface approximation via temporal extrapolation is proposed for devising decoupled marching algorithms for the mixed model. Under a time step limitation of the form $C_1h \leq \Delta t \leq C_2h$ (where h, Δt are mesh size and time step, respectively, and C_j, $j \in \{1, 2\}$, are two physical parameters) we prove both uniform and asymptotic stabilities over long time intervals. Some numerical experiments are presented and discussed.

Keywords: Non-stationary mixed Stokes-Darcy model, interface coupling, Crank-Nicolson scheme, explicit MacCormack scheme, MacCormack rapid solver method, long time stability.

AMS Subject Classification (MSC). 65N15, 65N30, 76D07, 76S05.

1 Introduction

The computational fluid dynamics (CFD) "frontier" has advanced from the simple to the complex. Generally, the simple methods taxed the available computational power when they occupied the frontier. The evaluation proceeded from methods for various forms of the potential and Navier-Stokes equations, or Stokes equations in the surface region to the Darcy's law in the subsurface region and then to non-stationary mixed Stokes-Darcy model (for example, see [1], chapters 6-8, and [3, 9, 10, 28, 11]) which is the subject of this work. Most of the schemes were developed at a time when the use of the Navier-Stokes equations was prohibitive for many problems because of the large computer memory or CPU time required. If the partitioned methods for such evolutionary problems were considered economical of computer resources when they were introduced, they are still so [23, 19, 17]. In this work, we consider the coupled fluid flow and porous media flow modeled by a mixed Stokes-Darcy problem. As literature on the mathematical analysis, numerical methods, and applications for the evolutionary groundwater-surface water flows, see for example [23, 19, 17, 2, 9, 10, 11, 12, 15, 16, 18, 20, 25] and the references therein.

To specify the problem considered, let Ω_f be a fluid flow domain coupled with a porous media flow in Ω_p and lie across an interface Γ from each other, where $\Omega_{f/p} \subset \mathbb{R}^d$ (d=2 or 3) are bounded domains, that is, $\Omega_f \cap \Omega_p = \emptyset$ and $\Gamma = \overline{\Omega}_f \cap \overline{\Omega}_p$. Designating by $\overline{\Omega} = \overline{\Omega}_f \cup \overline{\Omega}_p$, n_f and n_p the unit outward normal vectors on $\partial\Omega_f$ and $\partial\Omega_p$, respectively, and τ_j, $j = 1, ..., d-1$, the unit tangential vectors on the interface Γ. It is worth noticing to recall that $n_p = -n_f$ on Γ.

Let T be a positive quantity. The fluid velocity and the porous media flow are governed by the Stokes equations and the equations given in [3, 28], respectively, that is

$$\frac{\partial u}{\partial t} - \nu\triangle u + \nabla p = f, \quad \text{in } \Omega_f \times [0, T], \tag{1}$$

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