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Identification of the reaction coefficient in time fractional diffusion equations

Xiaoyan Song* Guang-Hui Zheng† Lijian Jiang‡

ABSTRACT

In this paper, we present an inverse problem of identifying the reaction coefficient for time fractional diffusion equations in two dimensional spaces by using boundary Neumann data. It is proved that the forward operator is continuous with respect to the unknown parameter. Because the inverse problem is often ill-posed, regularization strategies are imposed on the least fit-to-data functional to overcome the stability issue. There may exist various kinds of functions to reconstruct. It is crucial to choose a suitable regularization method. We present a multi-parameter regularization $L^2 + BV$ method for the inverse problem. This can extend the applicability for reconstructing the unknown functions. Rigorous analysis is carried out for the inverse problem. In particular, we analyze the existence and stability of regularized variational problem and the convergence. To reduce the dimension in the inversion for numerical simulation, the unknown coefficient is represented by a suitable set of basis functions based on a priori information. A few numerical examples are presented for the inverse problem in time fractional diffusion equations to confirm the theoretic analysis and the efficacy of the different regularization methods.

keywords: time fractional diffusion equation, reaction inversion, multi-parameter regularization

1 Introduction

Let Ω be an open bounded domain in $\mathbb{R}^2$ with a Lipschitz boundary $\partial\Omega$ and ν be the outward unit normal vector to $\partial\Omega$. Define $\frac{\partial u}{\partial \nu}$=$\nabla u \cdot \nu$. Let $T > 0$ be a fixed time length. Then we consider the time fractional diffusion equation(TFDE) with a reaction term as follows

$$\begin{cases} {}_0D_t^\alpha u(x,t) - \Delta u(x,t) + q(x)u(x,t) = 0 \;\text{ in }\; \Omega \times (0,T], \\ u(x,0) = 0 \;\text{ in }\; \Omega, \\ u(x,t) = \lambda(t)g(x) \;\text{ on }\; \partial\Omega \times (0,T]. \end{cases} \tag{1.1}$$

*College of Mathematics and Econometrics, Hunan University, Changsha 410082, China. Email: xiaoyansong@hnu.edu.cn
†College of Mathematics and Econometrics, Hunan University, Changsha 410082, China. Email: zhgh1980@163.com
‡School of Mathematical Sciences, Tongji University, Shanghai 200092, China. Email: ljjiang@tongji.edu.cn. Corresponding author