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Pricing European options under uncertainty with application of Levy processes and the minimal L^q equivalent martingale measure

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ABSTRACT

The study involves pricing of European options with usage of stochastic and fuzzy methods. We use a geometric Levy process as the model of the underlying asset, assuming that the log-price of a primary financial instrument is a jump–diffusion with jump part described by a linear combination of time-homogeneous Poisson processes. Analytical option pricing formulas in crisp case, using the minimal L^q equivalent martingale measure, are derived. The pricing expressions are achieved employing probability and stochastic analysis. The fuzzy counterparts of some model parameters are explored due to the fact that they are imprecisely evaluated. Applying fuzzy arithmetic, we derive the analytical option pricing expressions with fuzzy parameters. Moreover, we conceptualize a method of decision-making, taking into consideration the obtained fuzzy formulas. At last, we go through numerical examples to illustrate our theoretical results. Our main achievement in the paper is overcoming difficulties related to derivation of the option pricing formulas by advanced analysis of Jacod–Grigelionis characteristics of the log-price process used in our approach for the minimal L^q equivalent martingale measure as well as the skilled use of fuzzy arithmetic methods.

1. Introduction

Black and Scholes [1] achieved a historic breakthrough in the option pricing theory in complete markets. They used a geometric Brownian motion as the model of an underlying asset, obtaining the analytical European options pricing formulas (see, e.g. [2]). Although the Black–Scholes approach is convenient for analysis and computation, it suffers from various drawbacks or limitations. In the real world, the completeness of the market is rarely satisfied. Additionally, the distribution of log-return of an asset is often leptokurtic and skewed to the left (see [3,4]). Moreover, the smile property of implied volatility is a commonly known empirical phenomenon (see, e.g. [4–6]). In the financial literature concerning option pricing in incomplete markets, one can find many alternatives for the Black–Scholes approach, including log-price of an underlying asset described by Levy jump–diffusions with continuously distributed jumps (see [7] and references therein for further details). More general processes of an underlying asset are also widely considered, e.g. those describing evolution of energy prices, see [8].

The log-price model Y, proposed in this paper, is a Levy process, which is a sum of a drifted Brownian motion and a linear combination of Poisson processes, reflecting jumps in underlying asset prices. We derive the European options pricing

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formulas, applying probabilistic and stochastic techniques, in particular the martingale pricing method, as well as fuzzy set theory. The family of equivalent probability measures used for derivation of the European options pricing formulas is the family of minimal L^q equivalent martingale measures (for short ML^qEMM), where $q \in \mathbb{R} \setminus \{0, 1\}$ (see [4]). These measures (important for applications) appear when the utility function based martingale measures are considered (for further details see [4]). Moreover, Levy characteristic triplets (see, e.g. [9]) are extensively used in our approach.

The most important elements of fuzzy set theory (see [10]) used in this paper are the Extension Principle and fuzzy arithmetic. In real world, it is often unreasonable to choose or statistically estimate market and model parameters in the form of crisp numbers, since they fluctuate (see, e.g. [11]). The mentioned type of uncertainty is taken into account with application of fuzzy numbers. For this purpose, one can transfer experts' opinions into fuzzy numbers and introduce them to the pricing formulas. Similar method of obtaining the triangular fuzzy parameters was proposed for another financial application in [12]. In this paper, we also present and apply a method of decision-making based on the derived fuzzy options valuation expressions.

The author of [11] and [13] was the first to apply a fuzzy approach to option pricing in the case of the Black–Scholes model. Yoshida in [14] used the rational expected option price, depending on a fuzzy goal, as well as the geometric Brownian motion describing an underlying asset. The same stochastic model of a primary financial instrument was applied in [15] in the fuzzy reload option pricing problem. A jump–diffusion model for vulnerable options pricing and n-fold compound option pricing in fuzzy environment was used by the authors of [16] and [17], respectively. The option pricing problem in fuzzy framework was also considered in [18,19]. Other applications of stochastic and fuzzy methods can be found in [20,21]. Apart from financial applications, contemporary soft computing techniques are applied in other different fields (see e.g., [22,23,23–27]).

In this paper, similarly as in [7,28,29], we use a Levy process of jump–diffusion type for description of an underlying asset. We derive analytical option pricing formulas in crisp case, using the minimal L^q equivalent martingale measure for a fixed $q \in \mathbb{R} \setminus \{0, 1\}$. The obtained European options valuation expressions are the main contribution of this paper. To the best of our knowledge, they have not been published yet. Since the minimal variance equivalent martingale measure, considered in [7], is the minimal L^2 equivalent martingale measure, this paper should be regarded as a generalization and complement of the mentioned paper. Similarly as in [7], we describe the fuzzy option prices in a detailed way, using α-level sets of the model fuzzy parameters. We also propose a decision-making method, with applications of the derived fuzzy valuation expressions, as a useful tool for financial analysts. Furthermore, we present several numerical examples of market situations to illustrate our theoretical results. In particular, we conduct sensitivity analysis of the fuzzy option price with respect to selected parameters. We also suggest appropriate automatized recommendations for financial analysts, applying the proposed decision-making method. Finally, we investigate an influence of the parameter q (determining the ML^qEMM) on the price membership function and, as a consequence, on recommended investment decisions.

Main difficulties and challenges of our approach were related to derivation of the European option pricing formulas (19), (20) in the crisp case and the pricing formulas (26), (27), (29), (30) in the fuzzy case. Although the general theory of option pricing used in the paper was introduced by Miyahara (see Theorems 1, 2 and Remark 1), its application for the underlying asset model described by formulas (1) and (16) required advanced analysis of the Jacod–Grigelionis characteristics of the Levy process (16). In turn, difficulties with derivation of the fuzzy option pricing formulas were overcome by skilled use of fuzzy arithmetic, including Proposition 2.3 from [11], for each parameter $q \in \mathbb{R} \setminus \{0, 1\}$ of ML^qEMM.

As we have mentioned above, the problem of European option pricing was considered in our previous papers (see [7,28–31]). The log-price of the underlying asset, similarly as in this paper, was a sum of a drift, a Brownian part, and a linear combination of time-homogeneous Poisson processes. In [28,30], the jump part of the log-price process consisted of one and two elements, respectively. The valuation formulas of the financial derivatives depend on the underlying asset process and the equivalent probability measure, which was the minimal entropy equivalent martingale measure in [28,30,31], mean correcting and Esscher transformed martingale measure in [29], the minimal variance equivalent martingale measure (for short MVEMM) in [7], and the ML^qEMM in this paper. Derivation of the option valuation formulas for a chosen type of the equivalent martingale measure is a separate problem and it requires application of some elements of the theory of option pricing and stochastic analysis dedicated to this type of measure. Thus, the novel contribution of this paper is derivation of option pricing formulas in the crisp case for the ML^qEMM as well as their fuzzy counterparts in the analytical form. Such an approach, applied only in [7], enables to obtain the fuzzy option prices without application of Monte Carlo simulations, used in our other papers. Finally, the automatized method of decision-making was applied the first time to another problem in [32]. In this paper, similarly as in [7], we use this method in order to illustrate the usability of the fuzzy option pricing formulas to financial decision-making.

The paper is organized as follows. Notations and preliminaries on fuzzy arithmetic and stochastic analysis, used in the further part of the paper, are presented in Section 2. Section 3 is devoted to derivation and proof of the European call and put option valuation expressions in the crisp case. The fuzzy counterparts of the crisp valuation expressions are derived in Section 4. A method of financial decision-making is described in Section 5. In Section 6 we present and analyse numerical examples to illustrate our theoretical results. The last section contains conclusions.

2. Preliminaries

In this section we introduce necessary notations.

For details concerning definition and properties of fuzzy numbers as well as fuzzy and interval arithmetic we refer the reader to [7,11,33].