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Parallel accelerated cyclic reduction preconditioner for three-dimensional elliptic PDEs with variable coefficients

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Abstract

We present a robust and scalable preconditioner for the solution of large-scale linear systems that arise from the discretization of elliptic PDEs amenable to rank compression. The preconditioner is based on hierarchical low-rank approximations and the cyclic reduction method. The setup and application phases of the preconditioner achieve log-linear complexity in memory footprint and number of operations, and numerical experiments exhibit good weak and strong scalability at large processor counts in a distributed memory environment. Numerical experiments with linear systems that feature symmetry and nonsymmetry, definiteness and indefiniteness, constant and variable coefficients demonstrate the preconditioner applicability and robustness. Furthermore, it is possible to control the number of iterations via the accuracy threshold of the hierarchical matrix approximations and their arithmetic operations, and the tuning of the admissibility condition parameter. Together, these parameters allow for optimization of the memory requirements and performance of the preconditioner.

Keywords: Preconditioning, Cyclic reduction, Hierarchical matrices.

1. Introduction

This work focuses on the iterative solution of large-scale block tridiagonal linear systems of equations that arise from the discretization of elliptic partial differential equations on structured grids. Specifically, we demonstrate a parallel and scalable preconditioner based on an approximate factorization generated by the cyclic reduction algorithm [1]. Cyclic reduction uses a sequence of Schur complement reduction steps, with each step eliminating half of the unknowns. While an exact cyclic reduction would result in prohibitively expensive dense matrix blocks, we exploit the data-sparsity of these resulting blocks by approximating them in a hierarchically low-rank form featuring log-linear storage. This work builds on [2, 3], where a fast direct solver was introduced based on the synergy of parallel cyclic reduction and hierarchical matrices, and named accelerated cyclic reduction (ACR).

Iterative methods are advantageous for large-scale scientific computing since they feature tractable complexity and scalability, but their convergence is problem dependent. Direct methods, in contrast, guarantee

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