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SPLITTING SCHEMES FOR THE STRESS FORMULATION OF THE INCOMPRESSIBLE NAVIER–STOKES EQUATIONS *

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Key words. Navier-Stokes, Stress formulation, Splitting schemes

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Abstract. This paper presents a novel approach to the Navier-Stokes equations which reformulates them in terms of a new tensor variable. In the first formulation discussed in the paper this variable is proportional to the gradient of the velocity field with the pressure added to the diagonal components. In the second formulation it is identical to the stress tensor. At first glance the resulting tensorial problem is more difficult than the problem in the primitive variables. However, if combined with a proper splitting, it yields locally one dimensional schemes with attractive properties, that are very competitive to the most widely used schemes for the formulation in primitive variables. In addition, it has an advantage if applied to fluid-structure interaction problems.

1. Introduction. In this paper we reformulate the incompressible Navier-Stokes equations in terms of a new tensor variable similar or identical to the usual stress tensor. Further, we consider several possible splitting schemes for the resulting formulations and demonstrate their unconditional stability in the case of the unsteady Stokes equations. These schemes are derived similarly to the schemes proposed in Vabishchevich [9] in case of parabolic problems, and Konovalov [8] in case of hyperbolic problems in elasticity, and can be traced back to the flux-splitting schemes proposed by Degtyarev and Favorskii [1, 2]. The idea behind such formulations is to derive an evolutionary equation for the gradient of the original unknown, which involves a grad-div operator that is further discretized by treating the off-diagonal components, involving mixed derivatives, explicitly. The subsequent splitting schemes resemble very much the direction splitting schemes for parabolic equations.

Such an approach to the Navier-Stokes equations is very attractive in the case of fluid-structure interaction problems since in that case both the fluid and the structure problems can be reformulated in terms of the same stress variable (for the reformulation of the elasticity problem the reader is referred to Konovalov [8]). The fluid-structure boundary conditions can therefore be easily satisfied since they transform into a condition for continuity of the stress. As demonstrated in remark 2.1, the discrete problems in both domains would have a very similar form that would allow for a relatively easy simultaneous solution.

The remainder of the paper is organized as follows. In the next section 2 we first present the new formulation in terms of a tensorial variable for the standard form of the Stokes equations, in which the stress-divergence term, taking into account the incompressibility constraint, is reduced to the Laplacian of the velocity minus the pressure gradient. Then we introduce the various splitting schemes for this formulation, and study their stability. This section contains the most detailed information about the new approach. In the next section 3, we briefly show how it can be applied in case of the so-called stress-divergence formulation of the Stokes equations. The resulting system of equations differs from the system derived in section 2, however, the main ideas can be easily extended to the new formulation. In

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