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Abdellatif Agouzal, Karam Allali, Siham Binna

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Numerical analysis of in-situ biodegradation model in porous media

Abdellatif Agouzal[1], Karam Allali[2], Siham Binna[2]
[1] University Lyon1, CNRS UMR 5208, Institute Camille Jordan
69100 Villeurbanne, France
[2] University Hassan II, Department of Mathematics, Faculty of Science and Technology,
PO Box 146, Mohammadia, Morocco

Abstract

In this paper a numerical analysis of in situ-biorestoration is presented. Improved stability and error estimates are derived for a $\mathbb{P}1$ finite-element method applied to coupled system of non linear partial differential equations modeling flow in porous media. These results improve upon previously derived stability and error estimates in two respects: first, a stability of the approximate solution is demonstrated in the case of a non linear diffusive flux $\lambda(u)\nabla u$ with weaker norm assumptions than before and dependence only on the initial conditions and time interval, and second, error estimates are optimal as in linear case. Extensions include the finite element approximation of flow field, described by Darcy's law under the stream function-vorticity formulation because of its multiple advantages. Finally, a numerical simulations for a correlation exponential plume are presented.

1 Introduction

During the last decades, the production of toxic waste is remarkably increased leading to topsoil and subsoil pollution. The contaminants are of diverse origins and can be either organic in nature, such as toluene, detergents...; inorganic such as nitrates, fluorite, radium; or radioactive. This sometimes reaches groundwater pollution, drinking water sources for the entire population. In recent years, in-situ biodegradation technique appears as a promising method for treatment of polluted sites, it is based on stimulating the natural activity of microorganisms in the biological environment by providing nutrient substrates. Indeed, different studies demonstrated the ability of certain kind of bacteria to degrade different types of pollutants in the presence of substrates as the oxygen, it is called then aerobic degradation; when oxygen is not present, other substrates such as nitrogen can be used, in this case degradation occurs in anaerobic environments.

Because in-depth studies on a field scale are expensive, mathematical modeling serves as a valuable link between the experimental studies to understand the mechanisms of interaction transport, sorption, and biodegradation, and analysing the sensitivity and significance of various parameters in model and providing into field data collection activities. A mathematical models proposed here is Borden and Bedient [1] or Wheeler [2, 6] which describe the aerobic restoration of a site polluted by a hydrocarbon substrate S in the presence of bacterium B. Experiments and measurements carried out on the site of UCC (united Greosoting company) on Texas are able to validate and determine the scope of application of the model [1]. We are then led to study a system of nonlinear and coupled equations that includes two parabolic transport-diffusion-reaction equations, modeling the transport and use of oxygen and pollutant, a differential equation that governs the development of bacteria and an elliptic equation derived from Darcy's law:

$$(\mathcal{P})\begin{cases} \phi R\dfrac{\partial S}{\partial t} - (\nabla(\lambda(S) + D(u)).\nabla S) + u.\nabla S + \phi F(Ox, S, B) = 0, \\ \phi\dfrac{\partial Ox}{\partial t} - (\nabla(\lambda(Ox) + D(u)).\nabla Ox) + u.\nabla Ox + \alpha\phi F(Ox, S, B) = 0, \\ \dfrac{\partial B}{\partial t} + mB - \delta F(Ox, S, B) = \sigma, \\ \kappa^{-1}\mu(S)u + \nabla p = f(S), \\ div\ u = 0, \end{cases}$$