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COMPARISONS OF THREE KINDS OF PLANE WAVE METHODS FOR THE HELMHOLTZ EQUATION AND TIME-HARMONIC MAXWELL EQUATIONS WITH COMPLEX WAVE NUMBERS *

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Abstract. In this paper we are concerned with some plane wave discretization methods of the Helmholtz equation and time-harmonic Maxwell equations with complex wave numbers. We design two new variants of the variational theory of complex rays and the ultra weak variational formulation for the discretization of these types of equations, respectively. The well posedness of the approximate solutions generated by the two methods is derived. Moreover, based on the PWLS-LSFE method introduced in [16], we extend these two methods (VTCR method and UWVF method) combined with local spectral element to discretize nonhomogeneous Helmholtz equation and Maxwell's equatons. The numerical results show that the resulting approximate solution generated by the UWVF method is clearly more accurate than that generated by the VTCR method.

Key words. Helmholtz equation, time-harmonic Maxwell's equations, well posedness, electromagnetic wave, plane wave basis, error estimates

AMS subject classifications. 65N30, 65N55.

1. Introduction. The plane wave method, which fall into the class of Trefftz methods [26], was first introduced to solve the Helmholtz equation. Examples of this approach include the Variational Theory of Complex Rays (VTCR) introduced in [22, 23], the Ultra Weak Variational Formulation (UWVF) (see [4, 5, 28]), the plane wave Lagrangian multiplier (PWLM) method [7, 25], the plane wave discontinuous Galerkin methods (PWDG) (see [9, 12, 31]) and the weighted plane wave least-squares (PWLS) method (see [14, 21, 29, 30, 27]). The plane wave discretization method have been extended to discretization of time-harmonic Maxwell equations recently (see [13, 15, 17]). The plane wave methods have an important advantage over the other methods for discretization of the Helmholtz equation and time-harmonic Maxwell equations: the resulting approximate solutions have higher accuracies, owing mainly to the choice of the basis functions satisfying the governing differential equation without boundary conditions.

Recently, the UWVF method was extended to solve Maxwell's equations in [17, 18]. The studies [17, 18] were devoted to computing the electric and magnetic fields in a non-absorbing medium or within the PML. It was pointed out in [17] (p.733) that *the permittivity of the material in the computational domain for Maxwell's equations is, in general, complex valued* (i.e., the material is an absorbing medium). Moreover, the study [17] provides a procedure based on the UWVF method to compute the fields in absorbing media. This procedure shows that a postprocessing step is needed in the case of an absorbing medium. In the recently published work [15], the PWLS method was extended to discretize the time-harmonic Maxwell equations in absorbing media, and the numerical results indicate that the approximate solution generated by the method is much more accurate than that generated by the UWVF method. Moreover, the VTCR method and PWDG method have not been extended to discretize the Helmholtz equation and time-harmonic Maxwell equations with complex

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