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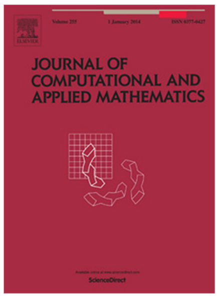

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Calculating the normalized Laplacian spectrum and the number of spanning trees of linear pentagonal chains*

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Abstract: Let W_n be a linear pentagonal chain with $2n$ pentagons. In this article, according to the decomposition theorem for the normalized Laplacian polynomial of W_n, we obtain that the normalized Laplacian spectrum of W_n consists of the eigenvalues of two special matrices: $\mathcal{L}_A$ of order $3n+1$ and $\mathcal{L}_S$ of order $2n+1$. Together with the relationship between the roots and coefficients of the characteristic polynomials of the above two matrices, explicit closed-form formulas for the degree-Kirchhoff index and the total number of spanning trees of W_n are derived, respectively. Finally, it is interesting to find that the degree-Kirchhoff index of W_n is approximately to one half of its Gutman index.

Keywords: Normalized Laplacian; Linear pentagonal chain; Resistance distance; Degree-Kirchhoff index; Spanning tree

AMS subject classification: 05C50

1. Introduction

Let $G=(V_G,E_G)$ be a graph with vertex set $V_G=\{v_1,v_2,\dots,v_n\}$ and edge set E_G. The set of neighbors of a vertex v in G is denoted by $N_G(v)$ or simply $N(v)$. Unless otherwise stated, we follow the traditional notations and terminologies (see, for instance, [2]).

The *adjacency matrix* $A(G)$ of G is a $|V_G|\times|V_G|$ matrix whose (i,j)-entry is equal to 1 if vertices v_i and v_j are adjacent and 0 otherwise. Let $D(G)=\mathrm{diag}(d_1,d_2,\dots,d_{|V_G|})$ be the diagonal matrix, where d_i is the degree of v_i in G for $1\leqslant i\leqslant|V_G|$. The (*combinatorial*) *Laplacian matrix* of G is defined as $L(G)=D(G)-A(G)$; see a nice survey paper [33] and two recent papers [34, 41].

Distance is an important quantity in graph theory (see [3]). On the one hand, this parameter effects the structure properties and algebraic properties of graphs; on the other hand, this parameter derives some other important distance-based parameters, such as average distance, diameter, radius, eccentricity, distance matrix, resistance distance, etc; see [18, 30, 35]. One famous distance-based parameter called the *Wiener index* [38], $W(G)$, was given by $W(G)=\sum_{i<j}d_{ij}$, where d_{ij} is the length of a shortest path connecting vertices v_i and v_j in G. For more conclusions and applications on the Wiener index, one may be referred to [14, 15]. The *Gutman index* of G was defined as $\mathrm{Gut}(G)=\sum_{i<j}d_id_jd_{ij}$ by Gutman in [19]. He also showed that when G is a tree of order n, the Wiener index and Gutman index are closely related by $\mathrm{Gut}(G)=4W(G)-(2n-1)(n-1)$.

On the basis of electrical network theory, Klein and Randić [29] proposed a novel distance function, namely the *resistance distance*, on a graph. Let G be a connected graph and we view G as an electrical network N by considering each edge of G as a unit resistor, then the resistance distance between vertices v_i and v_j, denoted by r_{ij}, is defined to be the effective resistance distance between v_i and v_j as computed with Ohm's law in N. This novel parameter is in fact intrinsic to the graph and has some nice interpretations and applications in chemistry

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