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A new approach for numerical solution of two-dimensional nonlinear Fredholm integral equations in the most general kind of kernel, based on Bernstein polynomials and its convergence analysis

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ABSTRACT

Regarding the efficient previous method for approximation of one-dimensional functions integrals through Bernstein polynomials in Amirfakhrian (2011), this paper presents a development of this method for approximation of two-dimensional functions integrals for the first time. Then, by combining this approximation with Bernstein collocation method for numerical solution of two-dimensional nonlinear Fredholm integral equations, the kernels double integrals of integral equations will be approximated. Combination of two-dimensional functions numerical integration method with numerical solution of integral equations method (in both methods, Bernstein polynomials were used) will result in increase of convergence speed and accuracy of the method. The convergence analysis of the method is completely presented. Finally, numerical examples are presented to illustrate the efficiency and superiority of our method in comparing it with other methods.

1. Introduction

Classical numerical solutions for different types of integral equations were previously investigated [1–3]. Numerical solutions of completely nonlinear and two-dimensional integral equations are interesting and usually difficult. As these equations are completely nonlinear, they include many types of integral equations. One and two-dimensional specific forms of these integral equations have been presented in numerous papers by the help of Bernstein polynomials. Numerical solution of one-dimensional nonlinear Volterra–Fredholm integro-differential equations was addressed by Bernstein collocation in [4]. Numerical solution of Volterra integral equations by Bernstein polynomial was the subject of [5]. Spline functions had crucial role in numerical solution of two-dimensional Fredholm integral equations in [6]. Operational two-dimensional Bernstein polynomial matrix has application in two-dimensional integral equations [7]. Bernstein polynomials have been used for solving various integral equation systems [8]. In [9,10], special forms of nonlinear two-dimensional Volterra–Fredholm integral equations were solved by Bernstein polynomials. An example of numerical solution for a special form of nonlinear two-dimensional Volterra equations by the help of Bernstein bases was presented in [11]. Some properties of two-dimensional Bernstein polynomials and their partial derivations were also calculated [12]. Bernstein bases can be

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applied in rational approximation for numerical solution of high-order boundary value problems [13]. One of the chapters in [14] is allocated to numerical solution for linear and nonlinear integral equations by Bernstein collocation method. A combination of Bernstein and Legendre methods or black pulse led to formation of a new method for numerical solution of integral equations [15,16]. Numerical solution of special form of nonlinear two-dimensional partial integro-differential equations with arbitrary order and nonlinear two-dimensional integral equations were investigated by Haar wavelets and collocation points [17,18]. Bernstein polynomials can be employed for numerical solution of linear integro-differential equations [19]. In physics, numerical solution of magneto-hemodynamic and electrohydrodynamic flow in a semi-porous channel has been carried out by Bernstein polynomials [20,21]. The rational Bernstein functions are among the tools for numerical investigation of velocity slip on unsteady flow over a surface. Numerical solution of special kind of nonlinear ordinary differential equations was performed [22]. Collocation Bernstein method was also used for simulating heat transfer of a micropolar fluid [23]. Bernstein polynomials are effective in presentation of semi-numerical method of off-centered stagnation flow towards a rotating disk [24].

Here, the numerical solution for following nonlinear two-dimensional Fredholm integral equation is addressed by two-dimensional Bernstein bases and the numerical integration techniques (presented in Section 3).

$$f(x,y) = g(x,y) + \int_0^1 \int_0^1 K(x,y,s,t,f(s,t))\,dsdt, \quad (x,y) \in [0,1] \times [0,1] \tag{1}$$

where g and K (kernel of integral equation) are known continuous functions defined on $[0,1]\times[0,1]$. $f(x,y)$ is an unknown real-valued function which will be determined.

The importance of solving these two-dimensional equations is their nonlinearity and generality. In Section 3, a new numerical integration technique is presented for two-dimensional functions; Section 4 describes the numerical solution method of this paper; while Section 5 provides a discussion on convergence analysis. Section 6 mentions some examples along with errors, error plots and comparison of this method with the previously-applied methods.

2. Preliminaries and Bernstein polynomials

2.1. One-variable Bernstein polynomials

One-variable Bernstein polynomials of degree m on the interval $[0, 1]$ are defined by

$$p_{(i,m)}(x) = \binom{m}{i} x^i (1-x)^{m-i}, \qquad x \in [0,1]$$

where

$$\binom{m}{i} = \frac{m!}{i!\,(m-i)!}, \quad m \in N.$$

Suppose that $H = L^2[0,1]$ is a Hilbert space with the inner product and $Y = \{p_{(0,m)}(x), p_{(1,m)}(x), \ldots, p_{(m,m)}(x)\}$ is a finite dimensional and closed subspace, therefore Y is a complete subspace of H. So, if f is an arbitrary element in H, it has a unique best approximation out of Y such as y_0, that is

$\exists y_0 \in Y$, st $\forall y \in Y$, $\|f - y_0\|_2 \leq \|f - y\|_2$, where $\|f\|_2 = \sqrt{\langle f, f\rangle}$, $\langle f, g\rangle = \int_0^1 f(x)g(x)dx$, [25]. So there exist unique coefficients $z_0, z_1, \ldots, z_m$ such that

$f(x) \simeq y_0 = \sum_{i=0}^{m} z_i p_{(i,m)}(x) = \mathbf{Z}^T \boldsymbol{\Phi}(x)$, where $\mathbf{Z}^T = [z_0, z_1, \ldots, z_m]$ and $\boldsymbol{\Phi}(x) = [p_{(0,m)}(x), p_{(1,m)}(x), \ldots, p_{(m,m)}(x)]^T$. (See [7].) The Bernstein's approximation $B_m(f(x))$ to a function $f(x) : [0,1] \to R$ is the polynomial

$$B_m(f(x)) = \sum_{i=0}^{m} f\left(\frac{i}{m}\right) p_{(i,m)}(x).$$

2.2. Two-variable Bernstein polynomials

Two-variable Bernstein polynomials of mnth degree are defined as follows.

$$p_{(i,m),(j,n)}(x,y) = \binom{m}{i}\binom{n}{j} x^i y^j (1-x)^{m-i} (1-y)^{n-j},$$

where $(x,y) \in [0,1] \times [0,1]$, $m, n \in N$, $i = 0, 1, \ldots, m$, $j = 0, 1, \ldots, n$, and m, n are arbitrary positive integers.

The Bernstein's approximation $f(x,y) : [0,1] \times [0,1] \to R$ is the polynomial

$$B_{m,n}(f(x,y)) = \sum_{i=0}^{m} \sum_{j=0}^{n} f\left(\frac{i}{m}, \frac{j}{n}\right) p_{(i,m),(j,n)}(x,y).$$

The two-variable Bernstein polynomials on $[0,1] \times [0,1]$ have the following properties: (see [7]):