Accepted Manuscript

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PII: S0377-0427(18)30338-8
DOI: https://doi.org/10.1016/j.cam.2018.05.061
Reference: CAM 11726

To appear in: *Journal of Computational and Applied Mathematics*

Received date : 4 January 2018
Revised date : 9 May 2018

ON APPROXIMATE INVERSE OF HERMITE AND LAGUERRE COLLOCATION DIFFERENTIATION MATRICES AND NEW COLLOCATION SCHEMES IN UNBOUNDED DOMAINS

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Abstract. In this paper, we provide an explicit, stable and fast means to compute the approximate inverse of Hermite/Laguerre collocation differentiation matrices, and also the approximate inverse of the Hermite/Laguerre collocation matrices of a second-order differential operator. The latter offers optimal preconditioners for developing well-conditioned Hermite/Laguerre collocation schemes. We apply the new approaches to various partial differential equations in unbounded domains and demonstrate the advantages over the usual collocation methods.

1. Introduction

The spectral methods are capable of providing very accurate simulation results with a relatively smaller number of degree of freedoms when compared with lower-order methods, so they are playing an ever increasingly important role in various computations related to a wide range of applications (see, e.g., [2, 3, 4, 22, 12, 33] and the references therein). Among several versions of spectral algorithms, the spectral collocation method is implemented straightforwardly in the physical space, and performs differentiation on a set of preassigned collocation points, where the equation under consideration is required to be satisfied, so it has remarkable advantages in dealing with variable coefficient and/or nonlinear problems. However, the practitioners are oftentimes plagued with ill-conditioning of the resulted linear systems.

It is known that the construction of suitable preconditioners is an effective means to circumvent this barrier. To date, considerable progress has been made in preconditioning spectral methods on finite domains. The successful attempts particularly include preconditioning usual collocation schemes by low-order finite difference or finite elements (see, e.g., [5, 6, 13, 14, 23, 24, 18]); and preconditioning differentiation by integration (see, e.g., [7, 16, 44, 19, 10, 11, 17, 21, 38, 40]). We highlight that Costabile and Longo [9] proposed the Birkhoff-Lagrange collocation methods using some interpolating basis polynomials associated with a suitable Birkhoff interpolation problem (cf. [27]) for boundary value problems (BVPs). Remarkably, Wang et al. [38] showed that such an approach led to well-conditioned collocation schemes and offered optimal preconditioners for usual collocation schemes. We refer to e.g., [39, 15, 46, 29, 8, 37] for recent followups and extensions of the Birkhoff collocation notion. It is important to remark that the method in [9, 38] can be thought of as the collocation analogue of the spectral-Galerkin method based on compact combinations of orthogonal polynomials [32, 20] in the sense that in both cases, the matrix corresponding to the

2000 *Mathematics Subject Classification.* 65N35, 65M70, 41A05, 41A10, 41A25.

Key words and phrases. Hermite and Laguerre differentiation matrices, new collocation schemes, unbounded domains.

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