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Rapid generalized Schultz iterative methods for the computation of outer inverses

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Abstract

We present a general scheme for the construction of new efficient generalized Schultz iterative methods for computing the inverse matrix and various matrix generalized inverses. These methods have the form $X_{k+1} = X_k p(AX_k)$, where A is $m \times n$ complex matrix and $p(x)$ is a polynomial. The construction procedure is general and can be applied to any number of matrix multiplications per iteration, denoted by θ. We use it to construct new methods for $\theta = 6$ matrix multiplications per iteration having (up to now) the highest computational efficiency among all other known methods. They are compared to several existing ones on a series of numerical tests. Finally, the numerical instability and the influence of roundoff errors is studied for an arbitrary generalized Schultz iterative method. These results are applicable to all considered new and existing particular iterative methods.

AMS Subject Classification: 15A09, 47J25.

Key words: Moore–Penrose inverse; Drazin inverse; Outer inverse; Iterative methods; Hyper–power methods; Convergence.

1 Introduction

Assume that $A \in \mathbb{C}^{m\times n}$ is a given matrix, while T and S are subspaces of $\mathbb{C}^n$ and $\mathbb{C}^m$, respectively, satisfying $AT \oplus S = \mathbb{C}^m$. In such a case, there exists a unique matrix $X = A^{(2)}_{T,S} \in \mathbb{C}^{n\times m}$ such that $XAX = X$, $\mathcal{R}(X) = T$ and $\mathcal{N}(X) = S$. Here, $\mathcal{R}(X)$ and $\mathcal{N}(X)$ denote the range and null–space of the matrix X. This unique matrix $X = A^{(2)}_{T,S}$ is known as *the outer inverse with prescribed range and null-space*. If $G \in \mathbb{C}^{n\times m}$ is a matrix such that $T = \mathcal{R}(G)$ and $S = \mathcal{N}(G)$, then the matrix $X = A^{(2)}_{T,S}$ is called *the outer G–inverse* or, simply, *the G–inverse*.

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