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Gibbs-Wilbraham oscillation related to an Hermite interpolation problem on the unit circle

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Abstract

The aim of this piece of work is to study some topics related to an Hermite interpolation problem on the unit circle. We consider as nodal points the zeros of the para-orthogonal polynomials with respect to a measure in the Baxter class and such that the sequence of the first derivative of the reciprocal of the orthogonal polynomials is uniformly bounded on the unit circle. We study the convergence of the Hermite-Fejér interpolants related to piecewise continuous functions and we describe the sets in which the interpolants uniformly converge to the piecewise continuous function as well as the oscillatory behavior of the interpolants near the discontinuities, where a Gibbs-Wilbraham phenomenon appears. Finally we present some numerical experiments applying the main results and by considering nodal systems of interest in the theory of orthogonal polynomials.

Keywords: Hermite-Fejér interpolation; approximation; Baxter class; para-orthogonal polynomials; unit circle; Gibbs-Wilbraham phenomenon.
2000 MSC: 41A05, 65D05, 42C05.

1. Introduction

Hermite interpolation problems on the unit circle have been thoroughly studied in the last years. In [1] it is given a method to determine the Laurent polynomials of Hermite interpolation in an efficient way. There the nodes are equally spaced on the unit circle and the method is based on the use of the FFT. Later on, in [3] the authors present two different ways of obtaining Laurent polynomials of Hermite interpolation on the unit circle by taking as nodal system the n roots of a complex number with modulus 1. One of them is based on a functional series whose coefficients can be computed efficiently by means of the FFT and the other relies on a barycentric formulation.

In [4] it was obtained nice expressions for the so called fundamental polynomials of the first and the second kind to express the Laurent polynomials of Hermite interpolation. There, the nodes are arbitrary complex numbers with modulus 1 and in the same paper, it was considered the case when the nodal polynomials are the para-orthogonal polynomials with respect to measures in the Szegő class and having Szegő function with analytic extension outside the disk. Under these conditions it was

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