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Convergence and non-negativity preserving of the solution of balanced method for the delay CIR model with jump

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ABSTRACT

In this work, we propose the balanced implicit method (BIM) to approximate the solution of the delay Cox–Ingersoll–Ross (CIR) model with jump which often gives rise to model an asset price and stochastic volatility dependent on past data. We show that this method preserves non-negativity property of the solution of this model with appropriate control functions. We prove the strong convergence and investigate the pth moment boundedness of the solution of BIM. Finally, we illustrate those results in the last section.

1. Introduction

Consider $(\Omega, \mathcal{F}, P)$ as a complete probability space with right continuous filtration $\{\mathcal{F}_t\}_{t\geq 0}$ while $\mathcal{F}_0$ contains all P-null sets. We consider the delay CIR model with jump introduced by Jiang, Shen and Wu [1]

$$\begin{cases} dS(t) = \lambda(\mu - S(t))dt + \sigma S(t-\tau)^{\gamma}\sqrt{S(t)}dW(t) + \delta S(t^-)d\tilde{N}(t), \quad t \geq 0, \\ S(t) = \xi(t), \quad t \in [-\tau, 0], \end{cases} \tag{1}$$

where γ, λ, μ and σ are positive constants, $W(t)$ is a standard Brownian motion and $\tilde{N}(t) = N(t) - \beta t$ is a compensated Poisson process, in which $N(t)$ is a Poisson process with intensity β and also $S(t^-) := \lim_{s\to t^-} S(s)$. The positive initial value ξ is an $\mathcal{F}_0$-measurable $C([-\tau, 0]; \mathbb{R}^+)$-valued random variable satisfying

$$E[\sup_{-\tau\leq t\leq 0} |\xi(t)|^p] < +\infty, \tag{2}$$

for any $p > 0$. In particular, the CIR model (1) without jump and delay, $\tau = 0$, $\delta = 0$, was introduced by Cox, Ingersoll and Ross [2], as a model for stochastic volatility, interest rate and other financial quantities. Also, the CIR model (1) without jump, $\delta = 0$, was introduced by Wu, et al. [3] with regard to the fact that stock prices depend on past behaviors. (See also [4,5].) Unfortunately, SDDEs with jumps have no explicit solution. Thus, constructing an appropriate numerical method

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to approximate and study the properties of the true solutions of these models is essential. Furthermore, in recent years, researchers are interested in numerical methods satisfying the same properties of the solutions such as positivity.

Strong convergence for stochastic differential equations (SDEs) with jumps is studied in the literature [6–11] and strong convergence for the mean-reverting square root process with jump is discussed in [12]. There are also some works concerned with positivity of numerical methods of SDEs; for example, see [13–19].

For the CIR model (1), with $\tau = 0, \delta = 0$, Dereich et al. [20] investigated the drift non-negativity preserving of implicit Euler method and in 2013, Higham et al. [21] introduced a new implicit Milstein scheme which preserves non-negativity of solution. Also, Halidias and Stamatiou in [22,23] constructed numerical schemes which preserve positivity of Heston 3/2 model and the CEV process, respectively. Lately, Yang and Wang [24] showed that the backward Euler scheme preserves positivity for the CIR model with jump.

In this manuscript, we choose a balanced implicit scheme in order to obtain the positivity of our approximation process. Non-negativity preserving of BIM for SDEs without jumps is well studied (see; e.g. [25,26]), and of SDEs with jumps is discussed in [27,24], under an appropriate choice of control functions. Also, Tan et al. [28] showed that the BIM preserves positivity for the stochastic age-dependent population equation.

Strong convergence for SDDEs with jumps is studied in the literature [29–34] and in [35] for SDDEs without jumps. Wu et al. [3], showed the existence of non-negative solution of the delay CIR model without jump and Jiang, et al. [1] proved it for the delay mean-reverting square root process with jump (1). Also, they showed the Euler Maruyama method converges strongly to the solution and proved the boundedness of the pth moments of the solution to the model and the method. Fatemioon et al. [36] investigated strong convergence of BIM for the CIR model and showed that the scheme preserves positivity.

To the best of our knowledge, there is no positivity preserving result of numerical method for SDDEs with jumps. The aim of this paper is to preserve positivity of BIM for SDDEs (the delay CIR models) with jumps (1). To do this, we cannot examine traditional control functions used in BIM for SDEs to reach the positivity of BIM for these SDDEs, for instance, see [27,14]. We define a new appropriate control function and prove that the positive solution of the BIM converges to the solution of the model (1) in the strong sense. Also we show the boundedness of p-moments of the method for any $p > 0$.

The paper is organized as follows. In Section 2, we propose the BIM for the SDDE with jump (1) and choose the appropriate control functions that imply the non-negativity property of the method. Also, we introduce the continuous case of the method to prove convergence results in the next section. In Section 3, we prove the convergence of the BIM applied to the model (1). Some numerical experiments in last section illustrate the obtained theoretical results of this paper.

2. Introduction of BIM and its properties

In this section, we describe the balanced method to approximate the solution of the delay CIR model with jump (1). Then, we state the non-negativity preserving concept of solution of numerical methods for this model, based on definitions in [26]. Also, we investigate the properties of p-moments for the balanced method in continuous time, which we need in the next section.

2.1. BIM and non-negativity preserving of method in discrete case

Set a uniform mesh on $[0, T]$, $t_n = nh, n = 0, \ldots, N, N \in \mathbb{N}$ for a step size $h \in (0, 1)$ as $h = \frac{\tau}{m}$, for a positive integer m. We introduce the BIM for SDDE with jump (1) by $s_n = \xi_n = \xi(t_n)$ for $n = -m, -m+1, \ldots, 0$ and for $n \geq 0$,

$$s_{n+1} = s_n + \lambda(\mu - s_n)h + \sigma s_{n-m}^{\gamma}\sqrt{s_n}\Delta W_n + \delta s_n \Delta\tilde{N}_n + C_n(s_n - s_{n+1}), \tag{3}$$

where $C_n = C_0(s_n, s_{n-m})h + C_1(s_n, s_{n-m})|\Delta W_n| + C_2(s_n, s_{n-m})\left|\Delta\tilde{N}_n\right|$, such that for control functions $C_0(s_n, s_{n-m})$, $C_1(s_n, s_{n-m})$ and $C_2(s_n, s_{n-m})$, the expression $(1+C_0(s_n, s_{n-m})h+C_1(s_n, s_{n-m})|\Delta W_n|+C_2(s_n, s_{n-m})\left|\Delta\tilde{N}_n\right|)^{-1}$ always exists and is uniformly bounded.

The control functions for the BIM (3) that ensure preserving non-negativity of the solution of delay CIR model with jump (1) are

$$C_0(s_n, s_{n-m}) = C_0 \geq \lambda, \tag{4}$$

$$C_1(s_n, s_{n-m}) = \begin{cases} \sigma s_{n-m}^{\gamma}\epsilon^{-\frac{1}{2}}, & s_n < \epsilon, \\ \sigma \dfrac{s_{n-m}^{\gamma}}{\sqrt{s_n}}, & s_n \geq \epsilon, \end{cases} \tag{5}$$

$$C_2(s_n, s_{n-m}) = C_2 \geq \delta, \tag{6}$$

where $C_0,\ C_2$ are positive constants.