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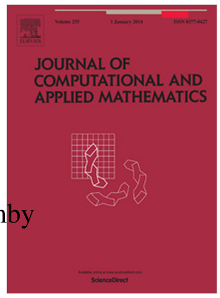

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Numerical approximation of stochastic evolution equations: Convergence in scale of Hilbert spaces

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Abstract

The present paper is devoted to the numerical approximation of an abstract stochastic nonlinear evolution equation in a separable Hilbert space H. Examples of equations which fall into our framework include the GOY and Sabra shell models and a class of nonlinear heat equations. The space-time numerical scheme is defined in terms of a Galerkin approximation in space and a semi-implicit Euler–Maruyama scheme in time. We prove the convergence in probability of our scheme by means of an estimate of the error on a localized set of arbitrary large probability. Our error estimate is shown to hold in a more regular space $V_\beta \subset H$ with $\beta \in [0, \frac{1}{4})$ and that the explicit rate of convergence of our scheme depends on this parameter β.

Keywords: Goy and Sabra shell model, nonlinear heat equation, Galerkin approximation, time discretization, fully implicit scheme, semi-implicit scheme, convergence in probability

1. Introduction

Throughout this paper we fix a complete filtered probability space $\mathfrak{U} = (\Omega, \mathscr{F}, \mathbb{F}, \mathbb{P})$ with the filtration $\mathbb{F} = \{\mathscr{F}_t; t \geqslant 0\}$ satisfying the usual conditions. We also fix a separable Hilbert space H equipped with a scalar product $(\cdot,\cdot)$ with the associated norm $|\cdot|$ and another separable Hilbert space $\mathscr{H}$. In this paper, we analyze numerical approximations for an abstract stochastic evolution equation of the form

$$\begin{cases} d\mathbf{u} = -[A\mathbf{u} + B(\mathbf{u},\mathbf{u})]dt + G(\mathbf{u})dW, \ t \in [0,T], \\ \mathbf{u}(0) = \mathbf{u}_0, \end{cases} \tag{1.1}$$

where hereafter $T > 0$ is a fixed number and A is a self-adjoint positive operators on H. The operators B and G are nonlinear maps satisfying several technical assumptions to be specified later and $W = \{W(t); 0 \leqslant t \leqslant T\}$ is a $\mathscr{H}$-valued Wiener process.

The abstract equation (1.1) can describe several problems from different fields including mathematical finance, electromagnetism, and fluid dynamic. Stochastic models have been widely used to describe small fluctuations or perturbations which arise in nature. For a more exhaustive introduction to the importance of stochastic models and the analysis of stochastic partial differential equations, we refer the reader to [18, 32, 37, 40, 42].

Numerical analysis for stochastic partial differential equations (SPDEs) has known a strong interest in the past decades. Many algorithms which are based on either finite difference or finite element methods or spectral Galerkin methods (for the space discretization) and on either Euler schemes or

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