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A new high-order numerical method for solving singular two-point boundary value problems

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Abstract

Recently, Goh et al. [28] proposed a numerical technique based on quartic B-spline collocation for solving a class of singular boundary value problems (SBVP) with Neumann and Dirichlet boundary conditions (BC). This method is only fourth-order accurate. In this paper, we propose an optimal numerical technique for solving a more general class of SBVP subject to Neumann and Robin BC. The method is based on high order perturbation of the problem under consideration. The convergence of the proposed method is analyzed. To demonstrate the applicability and efficiency of the method, we consider four numerical examples, three of which arise in various physical models in applied science and engineering. A comparison with other available numerical solutions has been carried out to justify the advantage of the proposed technique. Numerical result reveals that the proposed method is sixth order convergent, which in turn is two orders of magnitude larger than in [28].

Keywords: Singular boundary value problems; Optimal quartic B-spline collocation; Convergence analysis; Error estimation; Shallow membrane cap; Equilibrium of the isothermal gas sphere; Thermal explosion
2010 MSC: 65L10; 65L60; 34B16

1. Introduction

Consider the following SBVP:

$$y''(x) + \left(\frac{\alpha}{x} + \frac{p'(x)}{p(x)}\right)y'(x) = g(x, y(x)), \quad 0 < x \leq 1, \tag{1}$$

subject to the BC

$$y'(0) = 0, \qquad ay(1) + by'(1) = c. \tag{2}$$

Here $\alpha > 0$, $a > 0$, $b \geq 0$ and c are finite constants. The functions $g(x, y)$ and $p(x)$ satisfy the following conditions:

C1: $g(x, y)$ is continuous for all $x \in [0, 1], y \in \Re$,

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