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Second-order two-scale computational method for damped dynamic thermo-mechanical problems of quasi-periodic composite materials

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HIGHLIGHTS

- The SOTS solutions are constructed by the multiscale asymptotic analysis.
- The importance of developing the SOTS solutions is theoretically explained.
- The convergence result with an explicit rate for the SOTS solutions is obtained.
- A SOTS numerical algorithm is proposed to solve these problems effectively.

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ABSTRACT

In this paper, a novel second-order two-scale (SOTS) analysis method and related numerical algorithm are developed for damped dynamic thermo-mechanical problems of quasi-periodic composite materials. The formal SOTS solutions for these problems are constructed by the multiscale asymptotic analysis. Then we theoretically explain the importance of the SOTS solutions by the error analysis in the pointwise sense. Furthermore, the convergence result with an explicit rate for the SOTS solutions is obtained in the integral sense. In addition, a SOTS numerical algorithm is proposed to solve these problems effectively. Finally, some numerical examples verify the feasibility and effectiveness of the SOTS numerical algorithm we proposed.

1. Introduction

With the rapid development of materials science, composite materials have been widely used in engineering applications owing to their attractive physical properties. Due to the influence of preparation technology and fatigue damage, composite materials will have periodic geometric structures, but material parameters reflecting physical and mechanical properties of composite materials are no longer whole-periodic, but quasi-periodic [1–6]. Functionally gradient materials (FGM) are a class of typical quasi-periodic composite materials [5,6]. In engineering applications, these composites are usually served under complex thermo-mechanical environments. With rapid development of space aircraft, it is significant to understand the thermo-mechanical responses of the composites. Hence, the thermo-mechanical performances of composites have been a research hotspot of scientists and engineers [6–13]. As far as we know, some studies were performed on thermo-mechanical

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problems of the composites. However, most of these studies focused on one-way thermo-mechanical coupling problems, namely, only the thermal field affects the mechanical field [10–13]. Besides, some researchers were devoted to the two-way thermo-mechanical coupling problems, but their researches did not consider the damping effect of elastic dynamic equations and their research objects were periodic composite materials [6–9]. On the other hand, homogenization theory for damped dynamic equations with periodic structures was studied in [14–17]. But the above researches only focused on the theoretical analysis for the homogenization of damped dynamic equations. In order to accurately analyze the thermo-mechanical mechanism of quasi-periodic composite materials, it is meaningful to study damped dynamic thermo-mechanical problems of quasi-periodic composite materials.

The aim of this paper is to develop a SOTS analysis method and related numerical algorithm for damped dynamic thermo-mechanical problems of quasi-periodic composite materials. To our knowledge, the direct numerical computation for these multiscale problems needs a tremendous amount of computational resources to capture micro-scale behavior due to the sharp variation between different components of the composites [7,8,18]. In addition, the stability of numerical scheme for this strongly coupled hyperbolic and parabolic system is also a difficult problem to handle. From the point of view of theoretical analysis, the error estimate of SOTS solutions with an explicit convergence rate is hard to get due to lack of a prior estimate for damped wave equations with nonhomogeneous boundary condition [7,19,20]. In order to overcome these difficulties, we develop a novel SOTS method to solve these multiscale problems by integrating asymptotic homogenization method (AHM), finite element method (FEM) and finite difference method (FDM) together. On the other hand, we impose the homogeneous Dirichlet condition on auxiliary cell problems instead of the classical periodic condition. In this case, if the definition domain of governing equations of investigated multiscale problem is union of entire periodic cells, the SOTS solutions of this multiscale problem will automatically satisfy the boundary conditions on the boundary of the definition domain. At this case, the explicit convergence rate of SOTS solutions is easily obtained because the SOTS solutions will satisfy automatically the boundary condition of model equations.

The remainder of this article is organized as follows. In Section 2, the detailed construction of the SOTS solutions for damped dynamic thermo-mechanical problems of quasi-periodic composite materials is given by multiscale asymptotic analysis. In Section 3, the results of error analysis in the pointwise sense of first-order two-scale (FOTS) solutions and SOTS solutions are obtained, respectively. By comparing the error analysis of FOTS solutions and SOTS solutions in the pointwise sense, we theoretically explain the importance of SOTS solutions in capturing micro-scale information. Afterwards, an explicit convergence rate for the SOTS solutions is derived under some hypotheses. In Section 4, a SOTS numerical algorithm based on FEM and FDM is presented to effectively solve these multiscale problems. In Section 5, some numerical results are shown to verify the validity of our SOTS algorithm. Finally, some conclusions are given in Section 6.

For convenience, throughout the paper we use the Einstein summation convention on repeated indices.

2. Multiscale asymptotic analysis of model equations

Consider the model equations for damped dynamic thermo-mechanical problems of quasi-periodic composite materials as follows:

$$\begin{cases} \rho(\mathbf{x},\frac{\mathbf{x}}{\varepsilon})\dfrac{\partial^2 u_i^\varepsilon}{\partial t^2}+\mu(\mathbf{x},\frac{\mathbf{x}}{\varepsilon})\dfrac{\partial u_i^\varepsilon}{\partial t}-\dfrac{\partial}{\partial x_j}\Big(C_{ijkl}(\mathbf{x},\frac{\mathbf{x}}{\varepsilon})\dfrac{\partial u_k^\varepsilon}{\partial x_l}-\beta_{ij}(\mathbf{x},\frac{\mathbf{x}}{\varepsilon})(T^\varepsilon-\widetilde{T})\Big) \\ \qquad =f_i \ \text{ in } \Omega\times(0,T^*), \\ \rho(\mathbf{x},\frac{\mathbf{x}}{\varepsilon})c(\mathbf{x},\frac{\mathbf{x}}{\varepsilon})\dfrac{\partial T^\varepsilon}{\partial t}-\dfrac{\partial}{\partial x_i}(k_{ij}(\mathbf{x},\frac{\mathbf{x}}{\varepsilon})\dfrac{\partial T^\varepsilon}{\partial x_j})+\widetilde{T}\beta_{ij}(\mathbf{x},\frac{\mathbf{x}}{\varepsilon})\dfrac{\partial}{\partial t}(\dfrac{\partial u_i^\varepsilon}{\partial x_j}) \\ \qquad =h \ \text{ in } \Omega\times(0,T^*), \\ \mathbf{u}^\varepsilon(\mathbf{x},t)=\widehat{\mathbf{u}}(\mathbf{x},t),\quad T^\varepsilon(\mathbf{x},t)=\widehat{T}(\mathbf{x},t) \quad \text{on } \partial\Omega\times(0,T^*), \\ \mathbf{u}^\varepsilon(\mathbf{x},0)=\mathbf{u}^0(\mathbf{x}),\quad \dfrac{\partial \mathbf{u}^\varepsilon(\mathbf{x},t)}{\partial t}\Big|_{t=0}=\mathbf{u}^1(\mathbf{x}),\quad T^\varepsilon(\mathbf{x},0)=\breve{T}(\mathbf{x}) \quad \text{in } \Omega \end{cases} \tag{1}$$

where Ω is a bounded convex domain in $\mathbb{R}^{\mathcal{N}}(\mathcal{N}=2,3)$ with a boundary $\partial\Omega$; $\mathbf{u}^\varepsilon(\mathbf{x},t)$ and $T^\varepsilon(\mathbf{x},t)$ are undetermined displacement and temperature fields; $\widehat{\mathbf{u}}(\mathbf{x},t)$, $\widehat{T}(\mathbf{x},t)$, $\mathbf{u}^0(\mathbf{x})$, $\mathbf{u}^1(\mathbf{x})$ and $\breve{T}(\mathbf{x})$ are known functions; ε represents the characteristic periodic unit cell size; h and f_i are the internal heat source and the body force; $\widetilde{T}$ is the initial temperature when the composites are stress-free. $\rho(\mathbf{x},\frac{\mathbf{x}}{\varepsilon})$ and $c(\mathbf{x},\frac{\mathbf{x}}{\varepsilon})$ are the mass density and specific heat; $\mu(\mathbf{x},\frac{\mathbf{x}}{\varepsilon})$ is the dissipation damping coefficient; $k_{ij}(\mathbf{x},\frac{\mathbf{x}}{\varepsilon})$ is the second order thermal conductivity tensor; $\beta_{ij}(\mathbf{x},\frac{\mathbf{x}}{\varepsilon})$ is the second order thermal modulus tensor; $C_{ijkl}(\mathbf{x},\frac{\mathbf{x}}{\varepsilon})$ is the fourth order elastic tensor. According to the literatures [6–9], the existence and uniqueness of multiscale problem (1) can be confirmed because multiscale problem (1) owns the similar governing equations to multiscale problems in literatures [6–9]. In virtue of Faedo–Galerkin method in [21], we can give the complete proof of the existence and uniqueness of multiscale problem (1). However, it will add a lot of article length if we give a complete