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Numerical evaluation of highly oscillatory Bessel transforms[1]

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Abstract In this paper we mainly focus on numerical evaluation of highly oscillatory Bessel transforms. Based on the multiple integral, the schemes for computing this class of the transform are presented. By means of the bounds of special functions, we give the corresponding errors of the schemes. To demonstrate the efficiency and accuracy of the proposed methods, some test examples are shown.

Keywords: Oscillatory function, Bessel transforms, Multiple integral, Numerical quadrature

MSC: 65D32, 65D30

1 Introduction

In many areas of science and technology, we often encounter the problem of numerically computing Bessel transforms

$$I[f] = \int_a^b f(x)J_v(\omega x)dx, \tag{1.1}$$

for example, in astronomy, optics, electromagnetics, seismology image processing. Here f is a sufficiently smooth real-valued function, $J_v(\omega x)$ is Bessel function of the first kind and of order v, $\omega \gg 1$, and $0 \le a < b < \infty$ (see [1]-[5]). For $\omega \gg 1$, Bessel function $J_v(\omega x)$ slowly decays when the frequency ω increases, so that the integrand oscillates rapidly [6]-[14]. The highly oscillatory integrand makes some general numerical methods to be out of operation. Therefore, we have to resort to other high precision numerical methods to compute this class of transforms. Up to now, some special methods are obtained by integrating between zeros (see [16], pp. 118; [8], [17]) or the modified Clenshaw-Curtis method is proposed for $\int_0^1 f(x)J_v(\omega x)dx$ [9].

The first known numerical quadrature scheme for oscillatory integrals was developed in 1928 by Louis Napoleon George Filon [18]. Filon presented a *Filon method* for efficiently computing the oscillatory integrals of the forms

$$\int_a^b f(x)\sin(\omega x)dx \quad and \quad \int_0^\infty \frac{f(x)}{x}\sin(\omega x)dx. \tag{1.2}$$

As originally constructed, the method consists of dividing the interval into $2n$ panels of size h, and applying a modified Simpson's rule on each panel. In other words, the function f is interpolated at the endpoints and midpoint of each panel by a quadratic function. In each panel the integral becomes a polynomial multiplied by the oscillatory kernel $\sin(\omega x)$, which can be integrated in closed form. The infinite integral was computed using a series transformation. This method was

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