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Method for recovering boundary data in a two-dimensional Poisson equation on annular domain

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Abstract

In this paper, we consider a two-dimensional inverse boundary value problem for Poisson equation on annular domain, consisting of recovering boundary data on the inner boundary from temperature data on the outer circle. This problem is ill-posed in the sense of Hadamard, i.e., small errors in the data can lead to arbitrarily large perturbations in the solution. We establish an infinite singular value expansion for sought boundary data assuming noise-free temperature measurements. In the case of corrupted data, the truncated series is used as a regularized solution; i.e. ill-posedness is dealt with by filtering away high frequencies in the solution. The truncation parameter is determined by Morozov's discrepancy principle and an error estimate to quantify the accuracy of the computed approximate solution is derived. The proposed regularization method is illustrated by numerical simulations using synthetic data.

Keywords : Inverse boundary value problems, Poisson equation, Inverse heat transfer, singular value expansion, Morozov's discrepancy principle.

1 Introduction

We consider an inverse boundary value problem (IBVP) on an annular domain schematically shown in Fig. 1, specified by the Poisson equation in polar coordinates

$$\lambda_w \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \lambda_w \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2} = \Psi, \quad 0 < r_{\rm I} < r < r_{\rm E},\ 0 \leq \theta \leq 2\pi, \tag{1}$$

and the boundary conditions

$$\lambda_w \frac{\partial T}{\partial r}(r_{\rm E}, \theta) = \alpha(T_{\rm env} - T(r_{\rm E}, \theta)), \quad 0 \leq \theta \leq 2\pi, \tag{2}$$

$$-\lambda_w \frac{\partial T}{\partial r}(r_{\rm I}, \theta) = Q(\theta), \quad 0 \leq \theta \leq 2\pi. \tag{3}$$

In inverse heat conduction problems, λ_w denotes the wall thermal conductivity, $\Psi = \Psi(r)$ is a source function, α is the reciprocal of the overall heat transfer resistance between the outer boundary and the surrounding environment with temperature $T_{\rm env}$, and $Q(\theta)$ is the heat flux on the internal boundary. The boundary value problem (1)-(3) is referred to as the forward or direct problem. We are concerned with a Cauchy-like inverse problem for the Poisson equation (1)

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