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Recovery of the solute concentration and dispersion flux in an inhomogeneous time fractional diffusion equation

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Abstract

In this paper, we consider recovery of solute concentration and dispersion flux in an inhomogeneous time fractional diffusion equation. We prove that the considered problem is ill-posed, i.e. the solution does not depend continuously on the data. In order to obtain a regularized solution, we propose a truncation regularization method. The convergence estimates are established under some priori bound assumptions for the exact solution. We present three numerical examples to show efficiency of the method.

Keywords: time fractional inhomogeneous diffusion equation; regularization method; Caputo fractional derivative; Fourier transform

Mathematics subject Classification 2000: 35K05, 35K99, 47J06, 47H10.

1. Introduction

The mathematical description of diffusion has a long history with many different formulations including phenomenological models based on conservation of mass and constitutive laws, probabilistic models based on random walks and central limit theorems, microscopic stochastic models based on Brownian motion and Langevin equations and mesoscopic stochastic models based on master equations and Fokker-Planck equations. A fundamental result common to the different approaches is that the mean square displacement of a diffusing particle scales linearly with time. Some experimental measurements indicates that the mean square displacement of diffusing particles scales as a fractional order power law in time. In recent years a great deal of progress has been made in extending the different models for diffusion to incorporate this fractional diffusion. The tools of fractional calculus have proven very useful in these developments, linking together fractional constitutive laws, continuous time random walks, fractional Langevin equations and fractional Brownian motions. We refer the readers to [1] for a tutorial of standard and fractional diffusion processes. Derivation, applications and approximation of a time-fractional nonlinear diffusion equations can be found in [2]. Fractional calculus is generalization of the traditional calculus that leads to similar concepts and tools as standard differential calculus, but with a much wider applicability. They appear naturally in sciences area of physics, chemical engineering, biology, signal processing, electrical, control theory, finance, population dynamics, etc; we refer the readers to [3]-[9] and the references cited therein.

As it is known, fractional (nonlocal) diffusion equations replace the integer-order derivatives in space and time by their fractional analogues and they are used to model anomalous diffusion, especially

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