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PII: S0377-0427(18)30206-1
DOI: https://doi.org/10.1016/j.cam.2018.04.025
Reference: CAM 11608

To appear in: *Journal of Computational and Applied Mathematics*

Received date : 22 April 2017
Revised date : 28 December 2017

Recovery of ruin probability and Value at Risk from the scaled Laplace transform inversion

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Abstract

In this paper, we propose three modified approximations of the ruin probability and the inverse function of the ruin probability using the inversion of the scaled values of Laplace transform suggested by Mnatsakanov et al. [1]. The problem of evaluating numerically the tail-Value at Risk of an insurance portfolio is also discussed briefly. Performances of the proposed constructions are demonstrated via the graphs and tables using several examples.

Keywords: Smooth approximation of ruin probability, Rate of approximation; The scaled Laplace transform inversion; Value at Risk, Tail-Value at Risk

1 Introduction

In this article we consider the classical risk model with risk reserve process $\{R_t\}_{t\geq 0}$. A risk reserve process, as defined in broad terms, is a model for the time evolution of the reserves of an insurance company (see, for example, [2] and [3]). Let $R_0 = u > 0$ be the initial reserve at time $t = 0$. Assume that the company receives income from premiums at a constant rate, say p, per unit time. Claims are paid according to the aggregate loss process $S(t) = \sum_{k=1}^{N(t)} X_k$, where $\{N(t),\ t \geq 0\}$, the total number of claims, is a Poisson process with intensity $\lambda > 0$ and the individual claims, $X_1, X_2, \ldots$, are independent and identically distributed nonnegative random variables, independent of $N(t)$. At time t, the reserve of the company is $R_t = u + ct - S(t)$ and the time to ruin is $\tau(u) = \inf\{t \geq 0 : R_t < 0\}$.

A basic quantity of the classical risk model is the *safety loading* (or the *security loading*) parameter η defined as the relative amount by which the premium rate c must exceed the average claim amount per unit time, say ρ. To avoid certain ruin, an insurance company must ensure that $\eta > 0$ *always*. The probability of ultimate ruin $\psi(u)$ is the probability that the reserve ever drops below zero (or predefined threshold).

$$\psi(u) = \mathbb{P}\left(\inf_{t\geq 0} R_t < 0 | R_0 = u\right) = \mathbb{P}(\tau(u) < \infty).$$

Value at Risk (VaR) represents the *worst* possible monetary loss from a financial investment over a future time-period, e.g., one-day, one-week, one-month, etc has become the benchmark risk measure in today's financial world [4, 5, 6]. For a given time horizon, say t, and confidence level $p \in (0, 1)$, the VaR of a portfolio is the loss in *market value* over the time horizon t that is exceeded with probability $1 - p$. In Dempster et al. [4], several contributors discussed the applications of VaR in the evaluation of different types of financial risk - market (due to price changes), credit (due to counterpart default), liquidity (the risk of unexpectedly large and negative cash flow over a short period due to market imbalance), operational (risk of fraud, trading errors, legal and regulatory risk, and so on), etc. In [7],