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Superlinear convergence using block preconditioners for the real system formulation of complex Helmholtz equations

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Abstract

Complex-valued Helmholtz equations arise in various applications, and a lot of research has been devoted to finding efficient preconditioners for the iterative solution of their discretizations. In this paper we consider the Helmholtz equation rewritten in real-valued block form, and use a preconditioner in a special two-by-two block form. We show that the corresponding preconditoned Krylov iteration converges at a mesh-independent superlinear rate.

1 Introduction

Complex-valued Helmholtz equations arise in the modelling of various applied problems, for instance, when air is periodically compressed into some closed compartment, e.g., in a car. For the iterative solution of their discretization, standard preconditioning methods such as incomplete factorization or (algebraic) multigrid methods are not efficient, mainly due to the effect of high indefiniteness and large wave-numbers [8, 14], therefore more efficient iterative solvers are still of great interest. A lot of recent research has been devoted to preconditioners arising as the discretization of the so-called "complex shifted Laplace" problems, see, e.g., [7, 8, 9, 10, 13]. These preconditioners require, however, use of complex arithmetics and solution of a still somewhat involved preconditioner. In this note we modify the preconditoner so that it can be solved directly in real arithmetics and still preserve the favourable properties of convergence. Each application of the action of the preconditioner involves essentially the solution of only two standard elliptic problems.