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Conservative polytopal mimetic discretization of the incompressible Navier-Stokes equations

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Abstract

We discretize the incompressible Navier-Stokes equations on a polytopal mesh by using mimetic reconstruction operators. The resulting method conserves discrete mass, momentum, and kinetic energy in the inviscid limit, and determines the vorticity such that the global vorticity is consistent with the boundary conditions.

To do this we introduce a dual mesh and show how the dual mesh can be completed to a cell-complex. We present existing mimetic reconstruction operators in a new symmetric way applicable to arbitrary dimension, use these to interpolate between primal and dual mesh and derive properties of these operators.

Finally, we test both 2- and 3-dimensional versions of the method on a variety of complicated meshes to show its wide applicability. We numerically test the convergence of the method and verify the derived conservation statements.

1. Introduction

One of the most popular numerical methods for simulating viscous incompressible flow is the Marker-And-Cell (MAC) scheme by Harlow and Welch [1]. The MAC scheme, published over 50 years ago, is a staggered mesh method in which the incompressible Navier-Stokes equations are discretized in terms of the normal velocity components at the cell faces and pressure variables in the cell centers of a Cartesian mesh. The staggered positioning of the velocity variables allows for an efficient discretization of the divergence-free condition and leads to exact conservation of mass. Soon it was shown that the MAC scheme, besides mass and momentum, also conserves secondary quantities, namely, vorticity and, in the inviscid case, kinetic energy. Moreover, spurious oscillations, that have to be artificially suppressed for non-staggered colocated schemes, are absent for the staggered MAC scheme. In [2] it is stated that "nobody will dispute, that on Cartesian grids, computation of incompressible flows is best performed with the staggered scheme proposed by Harlow and Welch".

Many extensions of the MAC scheme have been proposed since its introduction. It has been extended to Delaunay triangulations [3, 4] and to curvilinear meshes [5, 6]. Moreover, it was shown

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