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Fast Computational Approach to the Delta Greek of Non-linear Black-Scholes Equations

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Abstract

In this paper, we consider a class of non-linear option pricing models. The focus is on the numerical investigation of Delta equation, where the unknown solution is the first spatial derivative of the option value. We construct and analyze monotone and sign-preserving finite difference schemes for the problems. Newton's and Picard's iterative procedures for solving the non-linear systems of algebraic equations are proposed. On this base, in order to improve the computational efficiency, we develop fast two-grid algorithms. Numerical experiments, using also Richardson extrapolation in time, are discussed in terms of accuracy, convergence and efficiency.

Keywords: Delta Greek, Delta equation, finite difference scheme, Newton method, Picard method, monotonicity, convergence, two-grid method, Richardson extrapolation

1. Introduction

The pricing and hedging of European options is a fundamental and relevant problem of modern finance. Following the classical theory due to Black, Scholes and Merton, see e.g. [27] an option in a stylized and idealized financial model can be priced by a solution $V = V(S,t)$, to the linear Black-Scholes parabolic equation

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + (r-q)S\frac{\partial V}{\partial S} - rV = 0, \quad S > 0, \quad 0 \le t \le T. \tag{1}$$

Here t is the time variable, $r > 0$ is the interest rate, $q \ge 0$ is the dividend yield rate, T is the maturity and $\sigma > 0$ is a constant historical volatility of the underlying asset price S. The linear Black-Scholes equation with a constant volatility σ has been derived under several restrictive assumptions, for example, zero transaction costs, perfectly replicated portfolio, frictionless, market completeness, etc. In the following years there have been several approaches to generalize this model, see e.g. [1, 2, 4, 5, 12, 13, 16, 20].

In this paper, we consider Black-Scholes equation (1), in which the volatility is assumed to be a function of the underlying asset S, the time t and Gamma of the option (the Greek Gamma is the second derivative V_{SS}) i.e

$$\sigma = \sigma\left(S, t, \frac{\partial^2 V}{\partial S^2}\right). \tag{2}$$

The motivation to target our research to solve the non-linear Black-Scholes equation (1) with the volatility of the form (2) arises from more realistic option pricing models in which one can take into account nontrivial

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