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A NOTE ON THE UNIFORM INF-SUP CONDITION FOR THE BRINKMAN PROBLEM IN HIGHLY HETEROGENEOUS MEDIA

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Abstract. In this short note we study the interpolation spaces based uniform inf-sup condition for Brinkman equation in highly heterogenous domains. The result is known to hold in the case of a uniform constant permeability. Our numerical construction and experiments show that it is not true for the Taylor-Hood and the Mini elements applied to problems with highly heterogeneous coefficients.

Key Words: Brinkman equation, high contrast, inf-sup condition, preconditioning.

1. Introduction. Consider a bounded, polygonal domain $\Omega \subset R^d$, $d = 2, 3$ with a Lipschitz boundary $\partial\Omega$. We study the following, steady Brinkman equation

$$
\begin{aligned}
-\nabla \cdot (\tilde{\mu}(x)\nabla \mathbf{u}) + \frac{\mu(x)}{K(x)}\mathbf{u} + \nabla p &= \mathbf{f} \text{ in } \Omega, \\
\nabla \cdot \mathbf{u} &= 0 \text{ in } \Omega, \\
\mathbf{u} &= \mathbf{0} \text{ on } \partial\Omega,
\end{aligned}
\tag{1.1}
$$

where $0 < K(x) \in L^\infty(\Omega)$ is the permeability coefficient of the medium, $0 \le \mu(x) \in L^\infty(\Omega)$ and $0 \le \tilde{\mu}(x) \in L^\infty(\Omega)$ are the dynamic and effective viscosities of the fluid, respectively. The Brinkman equation was originally proposed as a correction to Darcy's model for viscous flows in a highly porous media [9], and it has been since derived from the Stokes equation through homogenization [1], and has also been extended for the flows through complex fluid-porous-solid domains via penalty approach [2, 3]. The Brinkman equation is used in vast number of applications, such as computational fuel cell dynamics [25], groundwater and oil reservoir modeling, composite manufacturing [16] and heat pipes [22].

The Brinkman equation includes both Darcy's law and Stokes equation, which makes its analysis challenging. Formally, in the regions where $K(x) \gg 1$, (1.1) is in Stokes mode and when $\tilde{\mu} \ll 1$, the equation is close to Darcy's law. Since Stokes and Darcy equations are well-posed in different functional spaces, the discretization of the above Brinkman system must be robust with respect to possible extreme values of the coefficients. In [8] the authors propose and study optimal preconditioners for the algebraic system obtained from discretization of time dependent Stokes problems, so that $\tilde{\mu} = 1$ and $\mu/K(x) = 1/\Delta t$ with Δt being the time-mesh size. A vast literature has been devoted for various finite element approximation of (1.1). In [36], the authors study the piecewise constant $\tilde{\mu}$ and $\frac{\mu}{K}$ case and propose a modified $H(\mathrm{div})$ conforming spaces to obtain uniform bounds. Similar approach is taken in [28]. Codina et. al. consider stabilized formulation for constant coefficient case in [5]. A dual-mixed method is studied in [20], where the stress tensor, velocity and the deviatoric part of the gradient vector are the unknowns.

We are interested in a uniform well-posedness of the Brinkman problem in the case of highly and rapidly varying discontinuous permeability tensor $K(x)$. Such a general case includes highly anisotropic problems, models of flows in perforated domains and layered media, and many more that could be a useful tool for modeling and computer simulation. In this paper, we consider the case of *diagonal* tensor characterized by the permeability coefficient $K(x)$. An important quantity for such problems is the contrast κ_Ω of the medium defined as

$$
\kappa_\Omega := \frac{\max_{x\in\Omega} K(x)}{\min_{x\in\Omega} K(x)}.
$$

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