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High order well-balanced discontinuous Galerkin methods based on hydrostatic reconstruction for shallow water equations

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Abstract

In this paper, we introduce high order well-balanced discontinuous Galerkin methods for shallow water equations over non-flat bottom topography, which preserve the *lake at rest* steady state. To achieve the well-balanced property, we propose to construct the numerical fluxes based on the hydrostatic reconstruction idea and to be in combination with a novel source term approximation as well as a decomposition algorithm. Rigorous theoretical analysis and extensive numerical results all verify that the current methods maintain the well-balanced property. In addition, numerical results also indicate that the resulting methods enjoy the ability to accurately capture small perturbations of the *lake at rest* steady state and keep the genuine high order accuracy for smooth solutions.

Keywords: Shallow water equations; discontinuous Galerkin methods; hydrostatic reconstruction; source term; well-balanced property; high order accuracy

1. Introduction

In this paper, we are interested in constructing high order discontinuous Galerkin (DG) methods for shallow water equations. Numerically solving the shallow water equations has a wide range of applications in hydraulic and coastal engineering [1, 2]. Under the assumption of hydrostatic pressure distribution and the vertical acceleration of water particles neglected, the shallow water equations over a non-flat bottom topography can be derived by depth-integrating the Navier–Stokes equations [3] and have the following form:

$$\begin{aligned} &h_t + \nabla \cdot (h\mathbf{u}) = 0, \\ &(h\mathbf{u})_t + \nabla \cdot \left(h\mathbf{u} \otimes \mathbf{u} + \tfrac{1}{2} g h^2 \mathbf{I}\right) = -gh\nabla b, \end{aligned} \tag{1}$$

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