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FLUX FORMULATION OF PARABOLIC EQUATIONS WITH HIGHLY HETEROGENEOUS COEFFICIENTS

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Abstract. In this paper we study the flux formulation of unsteady diffusion equations with highly heterogeneous permeablity coefficients and their discretization. In the proposed approach first an equation governing the flux of the unknown scalar quantity is solved, and then the scalar is recovered from its flux. The problem for the flux is further discretized by splitting schemes that yield locally one-dimensional problems, and therefore, the resulting linear systems are tridiagonal if the spatial discretization uses Cartesian grids. A first and a formally second order time discretization splitting scheme have been implemented in both two and three dimensions, and we present results for a few model problems using a challenging benchmark data set.

1. Introduction

Models of compressible multiphase flow and transport through porous media contain a nonlinear parabolic equation called Richards equation (see for example [5] and [14]):

$$\frac{\partial b(u)}{\partial t} = \nabla \cdot [k(\boldsymbol{x}, u)\nabla u + \boldsymbol{g}(\boldsymbol{x}, u)] + f(\boldsymbol{x}, t),$$

with $b(u)$ being a monotonically increasing function of u. The most challenging difficulty in solving this problem is usually posed by the highly heterogeneous (eventually tensorial) coefficient $k(\boldsymbol{x}, u)$ that may vary by several orders of magnitude and the characteristic length scale of such variations can be very small as compared to the size of the domain of the problem. The resolution of such variations requires the use of extremely fine grids that leads to serious computational challenges. Therefore, a number of approaches have been proposed to circumvent this difficulty by computing analytically (in case of periodic or random variations) or numerically some upscaled coefficient that can represent the effect of the small scale variations on the large-scale problem. The numerical approaches that are predominant in case of porous media flows, can be divided in two main categories. The methods in the first category usually solve on each discretization cell a local elliptic problem with Dirichlet boundary conditions and devise from these solutions a new set of basis functions (in case of finite element approximations) or a new finite difference approximation to the second order operator, that can represent the effect of the small-scale variations of the coefficient onto the large-scale discretization of the equation. Examples of such methods are the multiscale Galerkin finite element method ([2, 6, 9, 11]), mixed multiscale finite element method (e.g. [1, 3]), mortar multiscale methods (e.g. [22]), variational multiscale method (e.g. [15]), and the multiscale finite volume method developed in [14, 16, 17]. The methods in the second category rely on the solution of local eigenvalue problems and use the eigenfunctions corresponding to the first several smallest eigenvalues in order to discretize the problem at the large scale (see e.g. [10, 12]). Such an approach works well if the coefficient $k(\boldsymbol{x}, u)$ does not essentially change in time since then the computation of the basis or the set of eigenfunctions can be performed once at the beginning of the computation

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