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# Parametric domain decomposition for accurate reduced order models: Applications of MP-LROM methodology

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ABSTRACT

The multivariate predictions of local reduced-order-models (MP-LROM) methodology, recently proposed by the authors Moosavi et al. (0000), uses machine learning based regression methods to predict the errors of reduced-order models. This study considers two applications of MP-LROM. First, the error model is used in conjunction with a greedy sampling algorithm to generate decompositions of one dimensional parametric domains with overlapping regions, such that the associated local reduced-order models meet the prescribed accuracy requirements. Once a parametric domain decomposition is constructed, any parametric configuration belongs to (at least) one of the partitions; the local reduced-order model associated with that partition approximates the full order model at the given parameters within an accuracy level that is estimated a-priori. The parameter domain decomposition creates a database of the available local bases, local reduced-order, and high-fidelity models, and identifies the most accurate solutions for an arbitrary parametric configuration. Next, this database is used to enhance the accuracy of the reduced-order models using: (1) Lagrange interpolation of reduced bases in the matrix space; (2) Lagrange interpolation of reduced bases in the tangent space of the Grassmann manifold; (3) concatenation of reduced bases followed by a Gram–Schmidt orthogonalization process; and (4) Lagrange interpolation of high-fidelity model solutions. Numerical results with a viscous Burgers model illustrate the potential of the MP-LROM methodology to improve the design of parametric reduced-order models.



## 1. Introduction

Many physical phenomena in science and engineering are investigated today using large-scale computer simulation models. The ever-increasing complexity of these high-fidelity models poses considerable challenges related to computational time, memory requirements, and communication overhead in a parallel environment. A popular approach to alleviate these challenges is to construct inexpensive surrogate (approximate) models that capture the most important dynamical characteristics of the underlying physical models, but reduce the computational complexity by orders of magnitude. Examples of surrogates include response surfaces, low resolution models, and reduced-order models.

Reduced-order modeling uses snapshots of high-fidelity model solutions at different times to extract a low-dimensional subspace that captures most of the high-fidelity solution energy. The reduced-order surrogate is obtained by projecting the dynamics of the high-fidelity model onto the low-dimensional subspace. This is usually achieved by orthogonal or oblique

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projections coined as Galerkin or Petrov–Galerkin methods where the solution is searched as a linear combination of the basis vectors. Since the Galerkin method is actual an elliptic approach, applying it to hyperbolic models must be done with careful consideration [1]. The reduced dimension leads to a considerable reduction in computational complexity at the expense of a decreased solution accuracy. A reduced-order model approximates well the high-fidelity solution at a given point of operation (e.g., for the model parameter values for which the snapshots have been taken), but becomes less accurate away from that point (e.g., when the high-fidelity model is run with different parameter values).

To be useful, reduced-order models must accommodate changes over the entire parametric space without losing their accuracy, simplicity and robustness. Reduced-order model (ROM) accuracy and robustness can be achieved by constructing a global basis [2,3], but this strategy generates large dimensional bases that may lead to slow reduced-order models. Moreover, for fluid flows, the Galerkin expansion with global modes presumes synchronized flow dynamics. Whereas this assumption is true for internal flows, it is not suited for transient shear flows with uni-directional'hyperbolic' convection of vortices [4]. Changes in the operational settings may lead to deformation of leading flow structures [5] especially if the model is characterized by bifurcations and multiple attractors. Approaches such as limiting the operational setting, extending the mode sets [6] and offline/online set adaptation address the issue of mode deformation.

In localization approaches, the reduced-order models are built offline and one is chosen depending on the current state of the system. Local approaches have been designed for parametric [7,8] or state spaces generating local bases for both the state variables [9,10] and non-linear terms [8,11]. Dictionary approaches [12,13] pre-compute offline many basis vectors and then adaptively select a small subset during the online stage. Error bounds for reduced-order approximations of parametrized parabolic partial differential equations are available in [14].

In this study, we employ machine learning regression models to guide the construction of parametric space decompositions for solving parametric partial differential equations using accurate local reduced-order models. Whereas the current methodologies are defined in the sense of Voronoi tessellation and rely on K-means algorithms, our approach delimitates subregions of the parametric space by applying an Artificial Neural Networks model to estimate the errors of reduced-order models following a parametric domain sampling algorithm.

Machine learning methodologies have been applied to predict and model the approximation errors of low-fidelity and surrogate models [15–17]. The multi-fidelity correction (MFC) approach [18–21] has been developed to approximate the low-fidelity models errors in the context of optimization. The reduced order model error surrogates method (ROMES) [22] seeks to estimate full errors from indicators such as error bounds and reduced-order residual norms. Both ROMES and MFC models predict the error of global reduced-order models with fixed dimension using univariate functions.

In contrast, the authors' multivariate predictions of local reduced-order-model method (MP-LROM) [23] proposes a multivariate model to compute the error of local reduced-order surrogates. A MP-LROM model based on Artificial Neural Network and a sampling algorithm are applied here to construct decompositions of the parametric domain for solving parametric partial differential equations using local reduced-order models that are accurate within an admissible prescribed threshold. The proposed strategy relies on a greedy algorithm that samples the vicinity of each parameter value used to generate a local reduced-order model and finds an open ball such that for all the parameters in the ball the error of the local reduced-order model is less than the desired threshold. The essential ingredient is the MP-LROM error model which approximates the error of reduced-order model. Then a greedy technique is used to sample the parametric domain and generates a feasible region where a specific local reduced-order model provides accurate solutions within a prescribed tolerance. The union of these feasible regions forms a decomposition of the parametric domain. Different thresholds lead to different domain decompositions. The current methodology is designed for one dimensional parametric spaces and it is applied to the viscous 1D-Burgers model. A decomposition for the viscosity domain is generated for various error thresholds. Once the decomposition is constructed there is no need to run the high-fidelity model again, since for each parameter value $\mu$ there exists a parameter $\mu_p$, and the associated reduced-order model (basis and reduced operators), whose solution error is accurately estimated a-priori. The dimension $K_{POD}$ of the local basis is usually small since it depends only on one high-fidelity model trajectory.

The decomposition leads to a database of available bases, local reduced-order models and high fidelity trajectory. This database can be used to generate more accurate reduced-order models for an arbitrary parametric configuration. Three different approaches are compared here; i.e., bases interpolation, bases concatenation, and high-fidelity model solutions combination. For the first method, we perform a Lagrangian interpolation of the bases in the matrix space [24], or linearly interpolate their projections onto some local coordinate systems [24,25]. The second method follows the idea of the spanning ROM introduced in [26], where a projection basis is created by concatenating some of the available bases for an arbitrary parameter. The third method interpolates the associated high-fidelity solutions and then extracts the singular vectors to generate a new basis and local reduced-order model.

The remainder of the paper is organized as follows. Section 2 reviews the construction of reduced-order surrogates for parametric high-fidelity models (where the dynamics depends on a set of model parameters). Section 3 introduces the new methodology for constructing decompositions of the parametric domain using MP-LROM error predictions. Then the potential of combining the existing information for generating more accurate reduced-order model is discussed. Section 4 presents the applications of the proposed methodologies to a viscous 1D-Burgers system. Conclusions are drawn in Section 5.

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