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On convergence analysis of particle swarm optimization algorithm

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Abstract Particle swarm optimization (PSO), a population-based stochastic optimization algorithm, has been successfully used to solve many complicated optimization problems. However analysis on algorithm convergence is still inadequate till now. In this paper, the martingale theory is applied to analyze the convergence of the standard PSO (SPSO). Firstly, the swarm state sequence is defined and its Markov properties are examined according to the theory of SPSO. Two closed sets, the optimal particle state set and optimal swarm state set, are then obtained. Afterwards, a supermartingale is derived as the evolutionary sequence of particle swarm with the best fitness value. Finally, the SPSO convergence analysis is carried out in terms of the supermartingale convergence theorem. Our results show that SPSO reaches the global optimum in probability. Moreover, the analysis on SPSO proves that the quantum-behaved particle swarm optimization (QPSO) is also a global convergence algorithm. The proof of the SPSO convergence in this work is new, simple and more effective without specific implementation.

Keywords: Particle swarm optimization; Convergence; Markov chain; Martingale theory

1. Introduction

Particle swarm optimization (PSO) was firstly proposed to model the intelligent behaviors of bird flocking [1]. As a population-based stochastic optimization algorithm, PSO has attracted tremendous research attention in the world resulting in a huge number of variants of the basic algorithm, comprehensive surveys of which can be found in [2, 3]. Nowadays, it has been well recognized as an efficient method for intelligent search and optimization.

Current research on PSO mainly focuses on algorithmic implementations, improvements and engineering applications and has revealed many interesting findings. However, a comprehensive mathematical explanation for the general PSO is still lacking. Among which, the stability analysis and convergence analysis of PSO are two key problems of great significance and should be investigated in depth. The PSO stability analysis concerns how the essential factors affect the particle swarm dynamics, and under what kind of conditions particle swarm converges to some constant positions. Clerc and Kennedy [4] carried out the stability analysis of the particle dynamics for the first time. Then a more generalized stability analysis of the particle dynamics was conducted by Kadirkamanathan *et al.* [5] using Lyapunov stability theorem. Van den Bergh [6] proved that particles could converge to a stable point. Recently Liu [7] analyzed the order-2 stability of PSO based on a weak stagnation assumption, where a new definition of stability was proposed and an order-2 stable region was obtained.

The PSO convergence analysis investigates whether a global optimal solution can be reached when particle swarm converges. Jiang *et al.* [8] presented a stochastic convergence analysis on the standard PSO (SPSO) by using stochastic process theory. Combining with finite element grid technique, Poli and Langdon [9] set up a discrete Markov chain model of the bare-bones particle swarm optimization. Cai *et al.* [10] developed an absorbing Markov process model of PSO. They

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