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Adaptive Aggregation on Graphs[☆]

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Abstract

We generalize some of the functional (hypercircle) a posteriori estimates from finite element settings to general graphs or Hilbert space settings. Several theoretical results in regard to the generalized a posteriori error estimators are provided. We use these estimates to construct aggregation based coarse spaces for graph Laplacians. The estimator is used to assess the quality of an aggregation adaptively. Furthermore, a reshaping based algorithm is tested on several numerical examples.

Keywords: graph Laplacian, graph aggregation, multilevel hierarchy, hypercircle error estimates, matching

1. Introduction

The hypercircle identity was first introduced in [21] to study approximations to elastic problems. Then the so-called hypercircle methods are established and studied in a posteriori error estimates for finite element methods (see, for example, [20], for a comprehensive survey of the main results). Some pioneering works on the hypercircle methods are [14, 6].

The functional a posteriori error estimates are established in [22] for general elliptic problems defined from dual operators between Banach and Hilbert spaces. The estimation is used, for example, in [24] to bound the conforming error in discontinuous Galerkin approximations of elliptic problems (see also [15, 23]). In [1] the hypercircle method is used to construct a posteriori error estimates for the obstacle problem (see also [18]). The formulation of the hypercircle identity and error estimates, however, arises naturally in Hilbert space settings and can be applied to graph Laplacians, a fact which we exploit in what follows.

A multilevel graph coarsening scheme based on matching (that is, collapsing adjacent vertices into aggregates) is studied in [11] and later used in several AMG methods. For example, the multigrid method proposed in [12] uses matching on the graph of the stiffness matrix to solve convection–diffusion equations. The AGMG method (AGgregation-based algebraic MultiGrid) in [17] employs a pairwise aggregation algorithm by matching which minimizes a strength function. In [5] matching techniques which optimize matrix invariants were studied. In our work we use matching to generate multilevel hierarchies for solving the graph Laplacian. We point out that other coarsening techniques exist in the literature, for example the compatible relaxation algorithm [2, 16, 10, 3].

In this paper combinatorial graphs are considered and we are interested in approximations of the associated graph Laplacian matrix from coarse subspaces. We propose Raviart-Thomas-like coarsening schemes for both the vertex and edge spaces defined on graph aggregations. It can be shown that the functional a posteriori error estimates naturally apply to this setting and we provide a short proof of the estimation, inspired by the works in [14, 6, 21, 22]. The estimator is minimized by an inter-leaved method to achieve reliable bound of the error [13]. Lastly we propose a reshaping algorithm that generates aggregations adaptively. This algorithm can be used together with aggregation coarsening methods [11] to form multilevel hierarchies for

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