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Resuspension and transport of fine sediments by waves over a thin layer of viscoelastic mud with erosion

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Abstract

A mathematical model for the one-dimensional mass transport describing the concentration evolution of suspended sediments over a viscoelastic mud layer in the presence of erosion is presented. Using a perturbation method, the problem is set in terms of the concentration of particles at the water-mud interface. Numerical results show considerable difference from the cases of rigid and non-erodible interfaces. A singular behaviour of the particles concentration is observed when the mud depth approaches a resonant value, associated with negative convection velocity.

Keywords: Water waves; viscoelastic mud; water-mud interface; particles concentration.

1 Introduction

The sediments motion in the bottom of a aquatic medium caused by a flow have been considered an important problem to be studied for coastal engineering and for marine geology applications [11, 7].

The seabed have been usually treated like a viscous or a viscoelastic mud. Gade [5] was the first to use the viscous model, also called a Newtonian model, to study the effect of a non-rigid and waterproof bottom over a wave surface. In that mathematical model, the upper layer is insviscid, the vertical acceleration is neglected and an approximation of shallow waves is adopted. Using those hypotheses, Gade [5] deduced a dispersion relation that gives an explicit expression for the wave number. However, because of the shallow wave aproximation, his model is restricted to the cases when, for the effective depth H', the inequality $H'/L < 20$ is valid, where L is the wave-length.

De Wit [3] modified Gade's model for the situation of a non-hydrostatic water inviscid layer over a tiny viscous mud layer compared with the wave-length. The vertical acceleration in the lower layer was again neglected, but in the upper layer it was considered. With these considerations, a new dispersion relation was presented.

In Dalrymple and Liu [2], three layers models were developed. The mud layer were considered as viscous in horizontal and vertical directions. The lower layer was assumed tiny, in the same order of magnitude of the boundary layer. The flow velocities in the water and in the mud layer were found and the wavenumber versus the dimensionless depth was obtained.

Ng [12] presented an asymptotic theory for waves propagating over the surface of a water layer over a viscous and homogeneous mud layer. The basic assumption is that the thickness of the mud layer and of the boundary layer in the interface are of the same order of magnitude of the wave amplitude over the water layer. These assumptions result in an parameter $\epsilon = ka \ll 1$ that was used to order the terms that should be taken into account in the equations that describe the problem. Some of the obtained results were the analytic solutions of the water and mud velocities coordinates and a dispersion relation. Besides that, they observed with some numerical results that the wave damping increase with the fluid viscosity.

Zhang and Ng [15] presented a perturbation analysis based on the motion equations in the Lagrangian form for the wave motion over a viscoelastic mud layer. This material was described as a *Voight body* [8], an analogue of the spring-dashpot system. The horizontal and vertical flow velocities besides the pressure