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Comparative study of methods of various orders for finding repeated roots of nonlinear equations

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ABSTRACT

In this paper we are considering 20 (families of) methods for finding repeated roots of a nonlinear equation. The methods are of order up to 8. We use the idea of basin of attraction to compare the methods. We found that 4 methods performed best based on 3 quantitative criteria.

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1. Introduction

There are many iterative methods for the solution of a single nonlinear equation [1,2]. Most are for simple roots and a few are for a repeated root. Here we are only interested in methods for repeated roots. In fact, we will not discuss derivative-free methods or methods with memory.

The usual technique of comparing a new method to existing ones, is by comparing the performance on selected problems using one or two initial points or by comparing the efficiency index (see [1]). In recent work, one can find a visual comparison, by plotting the basins of attraction for the methods. The idea of using basins of attraction appeared first in Stewart [3] and followed by the works of Amat et al. [4,5], and [6], Scott et al. [7], Chicharro et al. [8], Chun et al. [9–12], Cordero et al. [13], Neta et al. [14,15], Argyros and Magreñan, [16], Magreñan, [17] and Geum et al. [18–20] and [21]. In later works [11,12,22–24], we have introduced a more quantitative comparison, by listing the average number of iterations per point, the CPU time and the number of points requiring 40 iterations. We have also discussed methods to choose the parameters appearing in the method and/or the weight function (see, e.g. [25]). The only papers comparing basins of attraction for methods to obtain multiple roots are due to Geum et al. [18,19] and [20], Neta et al. [26], Neta and Chun [27–29], and Chun and Neta [30,31].

First we list the methods we consider here with their order of convergence (p), number of function- (and derivative-) evaluations per step (ν) and efficiency (I).

(1) A method of order 1.5 for **double** roots ($p = 1.5,\ \nu = 3,\ I = 1.1447$)
(2) Modified Newton's method (also known as Schröder's method) ($p = 2,\ \nu = 2,\ I = 1.4142$)
(3) Halley or Hansen–Patrick ($p = 3,\ \nu = 3,\ I = 1.4422$)
(4) Victory–Neta ($p = 3,\ \nu = 3,\ I = 1.4422$)

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(5) Neta (Chebyshev-based method) ($p = 3,\ \nu = 3,\ I = 1.4422$)
(6) Dong (4 methods) ($p = 3,\ \nu = 3,\ I = 1.4422$)
(7) Osada ($p = 3,\ \nu = 3,\ I = 1.4422$)
(8) Laguerre ($p = 3,\ \nu = 3,\ I = 1.4422$)

- Euler–Cauchy
- Halley
- Ostrowski
- Hansen–Patrick

(9) Chun and Neta ($p = 3,\ \nu = 3,\ I = 1.4422$)
(10) Chun–Bae–Neta ($p = 3,\ \nu = 3,\ I = 1.4422$)
(11) Li et al. (6 methods) ($p = 4,\ \nu = 3,\ I = 1.5874$)
(12) Kanwar et al. ($p = 4,\ \nu = 3,\ I = 1.5874$)
(13) Zhou et al. ($p = 4,\ \nu = 3,\ I = 1.5874$)
(14) Liu and Zhou ($p = 4,\ \nu = 3,\ I = 1.5874$)
(15) Sbibih et al. ($p = 4,\ \nu = 3,\ I = 1.5874$)
(16) Soleymani ($p = 4,\ \nu = 3,\ I = 1.5874$)
(17) Geum et al. ($p = 4,\ \nu = 3,\ I = 1.5874$).
(18) Geum et al. ($p = 6,\ \nu = 4,\ I = 1.5651$)
(19) Geum et al. ($p = 6,\ \nu = 4,\ I = 1.5651$)
(20) Geum et al. ($p = 8,\ \nu = 4,\ I = 1.6818$).

(1) A method of order 1.5 for **double** roots given by Werner [32]

$$\begin{aligned} y_n &= x_n - u_n, \\ x_{n+1} &= x_n - s_n u_n, \end{aligned} \tag{1}$$

where

$$s_n = \begin{cases} \dfrac{2}{1+\sqrt{1-4r_n}} & \text{if } r_n \le \dfrac{1}{4} \\ \dfrac{1}{2r_n} & \text{otherwise.} \end{cases}$$

We always use

$$u_n = \frac{f_n}{f_n'}, \tag{2}$$

$$r_n = \frac{f(y_n)}{f_n}, \tag{3}$$

and $f_n^{(i)}$ is short for $f^{(i)}(x_n),\ i = 1, 2, \ldots$.

Remark. We will not experiment with this method, since it is of a low order and limited to the case of double roots. One can see the basins for this method for the case of $(z^2-1)^2$ in [26].

(2) The quadratically convergent modified Newton's method is (see Schröder [33] or Rall [34])

$$x_{n+1} = x_n - m u_n. \tag{4}$$

(3) The cubically convergent Halley's method [35] which is a special case of the Hansen and Patrick's method [36]

$$x_{n+1} = x_n - \frac{u_n}{\frac{m+1}{2m} - \frac{u_n f_n''}{2f_n'}}. \tag{5}$$

(4) The third order method developed by Victory and Neta [37]

$$\begin{aligned} y_n &= x_n - u_n, \\ x_{n+1} &= y_n - \frac{f(y_n)}{f_n'}\frac{1+Ar_n}{1+Br_n}, \end{aligned} \tag{6}$$

where

$$\begin{aligned} A &= \mu^{2m} - \mu^{m+1}, \\ B &= -\frac{\mu^m(m-2)(m-1)+1}{(m-1)^2}, \\ \mu &= \frac{m}{m-1}. \end{aligned} \tag{7}$$