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Lawrence A. Harris

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Alternation points and bivariate Lagrange interpolation

Lawrence A. Harris

Department of Mathematics, University of Kentucky, Lexington, Kentucky 40506

Abstract

Given $m+1$ strictly decreasing numbers $h_0, h_1, \ldots, h_m$, we give an algorithm to construct a corresponding finite sequence of orthogonal polynomials $p_0, p_1, \ldots, p_m$ such that $p_0 = 1$, p_j has degree j and $p_{m-j}(h_n) = (-1)^n p_j(h_n)$ for all $j, n = 0, 1, \ldots, m$. Using these polynomials, we construct bivariate Lagrange polynomials and cubature formulas for nodes that are points in $\mathbb{R}^2$ where the coordinates are taken from given finite decreasing sequences of the same length and where the indices have the same (or opposite) parity.

Keywords: orthogonal polynomials, cubature, Christoffel-Darboux formula, Bézout identity
2010 MSC: 42C05, 65D05, 65D32

1. Introduction

Our object is to show that every decreasing finite sequence of real numbers is the set of alternation points of a finite sequence of orthogonal polynomials and to apply this to construct Lagrange polynomials and cubature formulas for the even and odd nodes of the Cartesian product of the points.

A motivating example of alternation points is the Chebyshev points $h_n = \cos(n\pi/m)$ and the corresponding polynomials are the Chebyshev polynomials T_n, where $T_n(\cos\theta) = \cos(n\theta)$. In previous papers [20, 24, 4], the alternation property (given in the Abstract) was used implicitly to construct two sets of bivariate polynomials having common zeros. The zeros were pairs of Chebyshev points where both indices of all pairs have the same or have opposite parity. We call two such sets of common zeros the even and odd product nodes, respectively.

Email address: `larry@ms.uky.edu` (Lawrence A. Harris)