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Iterative finite element variational multiscale method for the incompressible Navier-Stokes equations☆

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Abstract

Three iterative finite element variational multiscale methods are proposed and applied to the numerical simulation of the Navier-Stokes equations. The main idea of these methods is to combine the finite element variational multiscale method based on two local Gauss integrations with three different iterative schemes. The existence and uniqueness of approximate solutions of these iterative finite element variational multiscale methods are proved firstly, and then the convergence and error estimates of them are deduced. Finally, some numerical examples are given to support the theoretical analysis. The numerical results show that the iterative finite element variational multiscale method has a wider range of Reynolds numbers than standard Galerkin iterative finite element method, and the Oseen iterative scheme is much more efficient than the other two under high Reynolds numbers.

Keywords: iterative scheme, variational multiscale method, high Reynolds number, Navier-Stokes equations

2010 MSC: 65N55, 76D05, 76M10

1. Introduction

The incompressible Navier-Stokes equations are the fundamental partial differentials equations that describe the flow of the viscous Newtonian fluids, and the finite element method is one of the major tools used in numerical simulations of them[1, 2]. However, when solve the Navier-Stokes equations using the standard Galerkin finite element method at high Reynolds numbers, spurious numerical oscillations may occur in numerical results due to the domination of convection term. So it is challenging for constructing some stabilized finite element methods which are robust and efficient at a wide range of Reynolds numbers, especially at high Reynolds numbers.

There are numerous works on the stabilized finite element methods to overcome this defect, such as the defect-correction methods [3-6], the two-level and multi-level methods [7-11], the subgrid stabilized methods [12-16], and the techniques based on the variational multiscale method [17-19]. Among them,

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