

Accepted Manuscript

A parametric level set based collage method for an inverse problem in elliptic partial differential equations

Guangliang Lin, Xiaoliang Cheng, Ye Zhang

PII: S0377-0427(18)30079-7
DOI: <https://doi.org/10.1016/j.cam.2018.02.008>
Reference: CAM 11514

To appear in: *Journal of Computational and Applied Mathematics*

Received date: 4 October 2016
Revised date: 30 November 2017

Please cite this article as: G. Lin, X. Cheng, Y. Zhang, A parametric level set based collage method for an inverse problem in elliptic partial differential equations, *Journal of Computational and Applied Mathematics* (2018), <https://doi.org/10.1016/j.cam.2018.02.008>

This is a PDF file of an unedited manuscript that has been accepted for publication. As a service to our customers we are providing this early version of the manuscript. The manuscript will undergo copyediting, typesetting, and review of the resulting proof before it is published in its final form. Please note that during the production process errors may be discovered which could affect the content, and all legal disclaimers that apply to the journal pertain.



A parametric level set based collage method for an inverse problem in elliptic partial differential equations

Guangliang Lin^a, Xiaoliang Cheng^a, Ye Zhang^{b,c,*}

^a*School of Mathematics, Zhejiang University, Hangzhou, Zhejiang, 310027, China*

^b*School of Science and Technology, Örebro University, Örebro, SE-701 82, Sweden*

^c*Faculty of Mathematics, Technische Universität Chemnitz, D-09107 Chemnitz, Germany*

Abstract

In this work, based on the collage theorem, we develop a new numerical approach to reconstruct the locations of discontinuity of the conduction coefficient in elliptic partial differential equations (PDEs) with inaccurate measurement data and coefficient value. For a given conductivity coefficient, one can construct a contraction mapping such that its fixed point is just the gradient of a solution to the elliptic system. Therefore, the problem of reconstructing a conductivity coefficient in PDEs can be considered as an approximation of the observation data by the fixed point of a contraction mapping. By collage theorem, we translate it to seek a contraction mapping that keeps the observation data as close as possible to itself, which avoids solving adjoint problems when applying the gradient descent method to the corresponding optimization problem. Moreover, the total variation regularizing strategy is applied to tackle the ill-posedness and the parametric level set technique is adopted to represent the discontinuity of the conductivity coefficient. Various numerical simulations are given to show the efficiency of the proposed method.

Keywords: Inverse problem, Partial differential equations, Collage theorem, Regularization, Parametric level set, Total variation

1. Introduction

We are interested in reconstructing the locations of discontinuity of the conduction coefficient $\sigma(\mathbf{x}) \in L^\infty(\Omega)$ in the following elliptic differential equation

$$\begin{cases} -\nabla \cdot (\sigma \nabla u) = f & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases} \quad (1)$$

where Ω is a bounded closed domain in \mathbb{R}^N ($N \in \{1, 2, 3\}$) with piecewise smooth boundary $\partial\Omega$ and the source $f \in H^{-1}(\Omega)$ is given. The applications of the above inverse problem in the elliptic system can be found in many industrial problems, such as geophysical problems [1, 2], medical imaging [3], water resource problems [4], etc. If the coefficient value is unknown, some effective methods have been employed to find

*Corresponding author.

Email addresses: guanglianglin@zju.edu.cn (Guangliang Lin),
xiaoliangcheng@zju.edu.cn (Xiaoliang Cheng), ye.zhang@oru.se (Ye Zhang)

Download English Version:

<https://daneshyari.com/en/article/8901975>

Download Persian Version:

<https://daneshyari.com/article/8901975>

[Daneshyari.com](https://daneshyari.com)