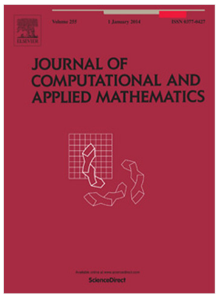

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A parametric level set based collage method for an inverse problem in elliptic partial differential equations

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Abstract

In this work, based on the collage theorem, we develop a new numerical approach to reconstruct the locations of discontinuity of the conduction coefficient in elliptic partial differential equations (PDEs) with inaccurate measurement data and coefficient value. For a given conductivity coefficient, one can construct a contraction mapping such that its fixed point is just the gradient of a solution to the elliptic system. Therefore, the problem of reconstructing a conductivity coefficient in PDEs can be considered as an approximation of the observation data by the fixed point of a contraction mapping. By collage theorem, we translate it to seek a contraction mapping that keeps the observation data as close as possible to itself, which avoids solving adjoint problems when applying the gradient descent method to the corresponding optimization problem. Moreover, the total variation regularizing strategy is applied to tackle the ill-posedness and the parametric level set technique is adopted to represent the discontinuity of the conductivity coefficient. Various numerical simulations are given to show the efficiency of the proposed method.

Keywords: Inverse problem, Partial differential equations, Collage theorem, Regularization, Parametric level set, Total variation

1. Introduction

We are interested in reconstructing the locations of discontinuity of the conduction coefficient $\sigma(\mathbf{x}) \in L^\infty(\Omega)$ in the following elliptic differential equation

$$\begin{cases} -\nabla \cdot (\sigma \nabla u) = f & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases} \tag{1}$$

where Ω is a bounded closed domain in $\mathbb{R}^N$ ($N \in \{1, 2, 3\}$) with piecewise smooth boundary $\partial\Omega$ and the source $f \in H^{-1}(\Omega)$ is given. The applications of the above inverse problem in the elliptic system can be found in many industrial problems, such as geophysical problems [1, 2], medical imaging [3], water resource problems [4], etc. If the coefficient value is unknown, some effective methods have been employed to find

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