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# Comments on the description and initialization of fractional partial differential equations using Riemann-Liouville's and Caputo's definitions

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**Abstract**

In this paper, it is shown that neither Riemann-Liouville's nor Caputo's definitions can be used to properly take into account initial conditions in the description of a fractional partial differential equation. This demonstration is done on a counterexample. Some suggestions are made to address this problem.

*Keywords:* Fractional partial differential equations, Caputo's fractional differentiation, Riemann-Liouville's fractional differentiation, initialization.

## 1. Introduction

In the field of fractional systems, many results have been obtained through straightforward extensions of existing results dedicated to integer systems. Sometimes these generalizations were proposed a little hastily and led to incorrect interpretations. While the various definitions of fractional differentiation that now coexist are all mathematically exact, their physical meaning remains obscure [1], [2], [3], [4], [5]. The way in which some take initial conditions into account remains debatable. For the first time, the non-equivalence of Riemann-Liouville's and Caputo's definitions in the presence of initial conditions or more accurately *history* was revealed in [6]. The question that immediately followed was: what happens when these definitions are used in a pseudo state space description of the form:

$$D^{\alpha}x(t) = Ax(t) + Bu(t), \qquad y(t) = Cx(t) + Du(t) \tag{1}$$