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Abstract

In this work we investigate a filtration process whereby particulate is deposited in the flow domain, causing the porosity of the region to decrease. The fluid flow is modeled as a coupled Stokes-Darcy flow problem and the deposition (in the Darcy domain) is modeled using a nonlinear equation for the porosity. Existence and uniqueness of a solution to the governing equations is established. Additionally, the nonnegativity and boundedness of the porosity is shown. A finite element approximation scheme that preserves the nonnegativity and boundedness of the porosity is investigated. Accompanying numerical experiments support the analytical findings.

Key words. Stokes equation, Darcy equation, filtration
AMS Mathematics subject classifications. 76505, 76D07, 35M10, 35Q35, 65M60, 65M55

1 Introduction

Applications of filtration abound in our everyday lives. From the routine activities such as: the preparation of espresso coffee in the morning [13], the water we drink from the faucet [29], and the car we drive to work [26]. To the less obvious but not less important such as: The absorption of nutrients in the small intestine [22], the cleansing of blood in the kidneys [21], and the prevention of postoperative infections [23]. All these phenomena rely on the separation of some solid from a fluid by means of a medium that is permeable to the fluid but (mostly) impermeable to the solid.

In this work we investigate a filtration process whereby particulate is deposited in the flow domain, causing the porosity of the region to decrease. The fluid flow is modeled as a coupled Stokes-Darcy flow problem and the deposition (in the Darcy domain) is modeled using a nonlinear equation for the porosity. (See Figure 1.) The model considered in this paper extends the work presented in [11] where the analysis was restricted to the filtration domain.

The addition of the upstream flow domain introduces into the model a different set of flow equations which must be suitable coupled across the interface (Γ) between the two subdomains. The coupling equations, given in (2.6) represent the conservation of mass across Γ (see (2.6a)) , the conservation of the normal component of stress across Γ (see (2.6b)) , and an equation (the Beavers-Joseph-Saffman condition (2.6c)) for the tangential component of the stress vector on the Stokes domain.

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