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A competition flow method for computing medial axis transform

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ABSTRACT

Medial axis computation (MAC) has many applications in computer-aided design, computer graphics, automated mesh generation, solid modeling, CNC tool path generation, robotic motion planning, bioinformatics, and shape analysis. There are three key issues that need to be further addressed on MAC of a NURBS(Non-Uniform Rational B-Spline) curve boundary, namely (1) computation of regular medial axis points; (2) efficient detection of the existence of a branch point where two branches intersect; and (3) convergence and correctness on computing the position of a branch point. This paper presents a competition flow method for medial axis computation of a planar simply-connected NURBS boundary. Different from previous methods which trace a Voronoi Diagram Tree (VDT) in depth-first order from a terminal point, the competition flow method (CFM) traces the corresponding tree in breadth-first order. It has three main contributions. Firstly, it utilizes an optimized method based on previous state-of-art algorithms for computing regular medial axis points, which can achieve a higher computational efficiency. Secondly, it implements a divide-and-conquer method in a natural way by proposing a simple tree technique, in which a VDT is divided into several simple tree structures of level l, $l = 0, 1, \ldots$, for competitive medial axis computation. Finally, by using the competition flow method, possible places where branch points are located, are intuitively and efficiently detected and bounded with high precision, which ensures the convergence and the correctness of the computation of the position of a branch point, and also ensures the correctness of the topological structure of the medial axis tree. Numerical examples show that the new method can achieve a better performance compared with several existing methods.

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1. Introduction

The Medial Axis (MA) of a closed planar region is the locus of the center of the maximal disk contained within its boundary. The maximal disk of an inner point on a MA touches with the boundary curve at two or more foot points, which are closest points on the boundary curve with the minimum distance equal to the radius of the maximal disk. It is also called a skeleton, which was first introduced by Blum [1] to describe biological shapes. The MA together with its radius function defines a Medial Axis Transform (MAT) which has many applications in computer-aided design [1–3], computer graphics [4], bioinformatics [4], robotics for motion planing [5], etc. In the general fields of shape analysis and solid modeling, including automated meshing, it is regarded as an essential step to obtain the MAT for a given geometry (or shape/domain), and efficient and robust techniques are therefore required [6]. Medial axis transform (MAT) has been

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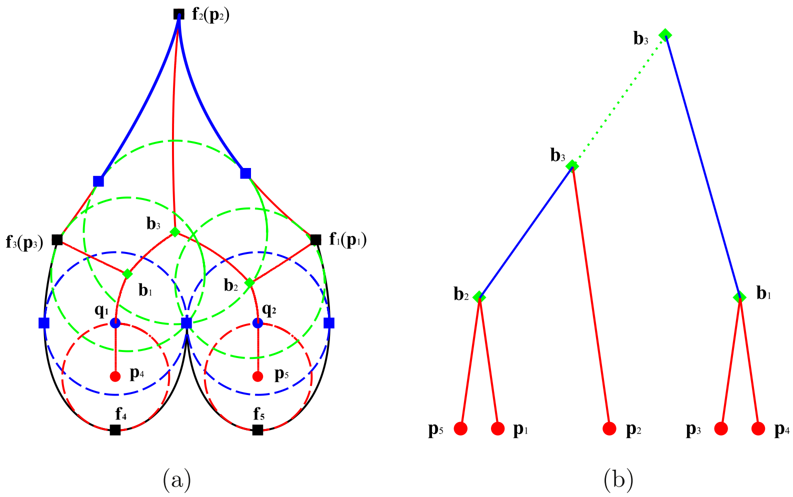

Fig. 1. (a) Illustration of some terminologies: (1) MA curve in red solid line; (2) foot points in solid box on the boundary curve; (3) terminal points in red dot, where terminal points $\mathbf{p}_i$ coincide with its foot point $\mathbf{f}_i$, $i = 1, 2, 3$; (4) regular points in blue dot, such as $\mathbf{q}_i$, $i = 1, 2$; (5) maximal disks denoted by dashed circle; (6) branch point in solid green diamond, such as $\mathbf{b}_i$, $i = 1, 2, 3$; and (7) foot area in solid blue which is part of the boundary, such as that of the branch $\mathbf{f}_2\mathbf{b}_3$; and (b) The Voronoi diagram tree: (8) leaf nodes, $\mathbf{p}_i$, $i = 1, 2, \ldots, 5$ in order; (9) non-leaf nodes $\mathbf{b}_i$, $i = 1, 2, 3$; and (10) virtual edge in dashed green. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

investigated for many years [4,7–35]. In the literature, there are mainly two models, i.e., grass-fire model for discrete MAT and the maximal disk model for continuous MAT [4,36,37]. The planar boundaries for the continuous MAT can be divided into three classes, i.e., polygon boundary [1,38], curvilinear polygon boundary [39,40] and NURBS boundary [4,5,8,13,18,41–44]. For polygon boundary and curvilinear polygon boundary, the corresponding branch points can be analytically expressed, and the algorithms mainly focus on the computational complexity or computational efficiency. For a NURBS boundary, an indirect method is to approximate it by using line segments, circular arcs and other conics within a given tolerance, and then the MA of the approximated boundary is computed as the resulting MA of the given NURBS boundary [8]. However, as pointed out in [15,18], the resulting MA of approximation boundaries may largely deviate from the ideal MA of the original boundary even though they fit the boundary very well.

This paper focuses on direct methods for computing the MA of a NURBS boundary, i.e., without approximating the boundary. Specifically, a competition flow method (CFM) is proposed for computing the medial axis transform from an input NURBS boundary. Numerical example show that the proposed method can be much more efficient and robust compared with several other existing methods.

The remainder of this paper is organized as follows. Section 2 provides some preliminaries with related work on the medial axis computation of NURBS boundaries. The main features of the proposed competition flow method is also highlighted at the end of this section. Section 3 presents the proposed competition flow method in details. In Section 4, comparisons between different methods are covered, and cases that NURBS boundaries have internal holes are also discussed. Final conclusions are drawn at the end of this paper in Section 5.

2. Preliminary and related work

2.1. Terminologies and properties

We introduce the following definitions, which are also illustrated in Fig. 1. In Fig. 1(a), there are five terminal points $\mathbf{p}_i$, $i = 1, 2, \ldots, 5$, following the counterclockwise ordering of their foot points $\mathbf{f}_i$ marked on the boundary. Note that there are no foot point of other branches between $\mathbf{f}_i$ and $\mathbf{f}_{i+1}$, it is said that the branches starting from $\mathbf{p}_i$ and $\mathbf{p}_{i+1}$ are adjacent, $i = 1, 2, 3, 4$; and similarly, branches starting from $\mathbf{p}_5$ and $\mathbf{p}_1$ are adjacent too. A regular point of a MA has two foot points, and the one which is on the left (or right) side when marching forward from a foot point or a branch point is called the left (or right) foot point. Fig. 1(b) illustrates the corresponding Voronoi diagram tree (VDT), in which the five terminal points are taken as its leaf nodes. From Property 1, a branch is prior to intersect with its adjacent branch, and we define branch pairs consisting of two branches prior to intersect with each other, such as $(\mathbf{p}_5, \mathbf{p}_1)$ and $(\mathbf{p}_3, \mathbf{p}_4)$ shown in Fig. 1(b). In principle, if a branch denoted by Ω_1 is adjacent to branch Ω_2, Ω_1 is said to be adjacent to arbitrary father branch of Ω_2.

Definition 1. A maximal disk is defined as a disk inside the given boundary which touches the boundary with at least one point and is not a proper subset of any other disk inside the boundary [43].

Definition 2. The MA of a boundary is defined as the locus of the center of the corresponding maximal disks [1,43].

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