Accepted Manuscript

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PII: S0377-0427(17)30620-9
DOI: https://doi.org/10.1016/j.cam.2017.12.006
Reference: CAM 11423

To appear in: *Journal of Computational and Applied Mathematics*

Received date : 30 June 2017
Revised date : 30 November 2017

An iterative numerical method for fractional integral equations of the second kind

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Abstract

In this paper we propose an iterative numerical method for approximating solutions of fractional integral equations of the second kind, based on Picard iteration and a suitable quadrature formula. Under certain conditions, we prove the existence and uniqueness of the solution and give error estimates of the approximations. Numerical examples are given, which illustrate the good approximations.

Keywords: Fractional integral equations, Abel integral equations, fixed-point theory, Picard iteration, numerical approximation, product integration.
2010 MSC: 26A33, 37C25, 45D05, 45E10, 65D30, 65R20.

1. Introduction

Integral equations represent an important field in the area of Applied Mathematics, as they are a powerful tool for modeling diverse problems arising in all areas of scientific research. Also, many initial and boundary value problems associated with ordinary and partial differential equations can be reformulated as integral equations.

Fractional calculus, which deals with derivatives and integrals of arbitrary order, has kept the interest of many researchers in recent years. Its use and applications in so many fields are virtually endless. Integral equations of fractional order – starting with Abel equations (of fractional order $\alpha = 1/2$) – have been studied extensively in the past few decades. They play an important role in modeling various phenomena in physics and engineering, such as heat conduction, diffusion, propagation of waves, radiative transfer, kinetic theory of gases, scattering in quantum mechanics, diffraction problems and water waves, radiation, continuum mechanics, potential theory, geophysics, electricity and magnetism, as well as in mathematical economics, communication theory, population genetics, queuing theory, or medicine. In recent years, ideas and methods from fractional calculus have been applied with great success to groundwater pollution and groundwater flow problems, or acoustic wave problems. A fractional model can be used for the phenomenology of non-local, non-diffusive transport processes observed in fusion plasmas, anomalous confinement time scaling, up-hill transport, pinch effects, and many more.

As for inverse boundary value problems, many initial and boundary value domains are fractal curves, which are continuous everywhere, but nowhere differentiable. Thus, classical calculus cannot be used to process ordinary or partial local fractional differential equations with fractal conditions, whereas fractional calculus can successfully handle fractals and continuous, but non-differentiable functions.

The solvability of such integral equations has been studied via many analytical and approximating methods. We mention existence (and uniqueness) results, under various conditions ([7, 6, 13, 21]), Laplace transforms ([15, 19]), pathway–type (P_α) transforms ([3]), fixed point theorems ([1, 2, 5]), homotopy perturbation transform method ([19]), etc. Numerical methods have been derived, based on collocation and iterated collocation ([11, 12, 14]), product integration ([8, 20]), quadrature schemes ([6]), combined regularization–Adomian decomposition methods ([18]), variational iteration methods ([22]), etc.

Many researchers have studied properties of the solutions, such as regularity ([11, 12, 20]), Ulam–Hyers stability ([4]), monotonicity ([13]), upper and lower bounds ([17]), Gronwall–type inequalities and continuous dependence on initial values ([23]) and others.

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