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A moving-mesh finite difference scheme that preserves scaling symmetry for a class of nonlinear diffusion problems

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Abstract

A moving-mesh finite difference scheme based on local conservation is presented for a class of scale-invariant second-order nonlinear diffusion problems with moving boundaries that (a) preserves the scaling properties and (b) is exact at the nodes for initial conditions sampled from similarity solutions. Details are presented for one-dimensional problems, the extension to multidimensions is described, and the exactness property is confirmed for two radially symmetric moving boundary problems, the porous medium equation and a simplistic glacier equation.

In addition, the accuracy of the scheme is also tested for non self-similar initial conditions by computing relative errors in the approximate solution (in the l_∞ norm) and the approximate boundary position, indicating superlinear convergence.

Keywords: Nonlinear diffusion, moving-meshes, scale-invariance, similarity, conservation, finite differences, porous medium equation, glacier equation, radial symmetry.

1 Introduction

Partial differential equations (PDEs) govern many physical processes that occur in branches of applied mathematics. However, due to the complexity of these equations the solution cannot always be determined analytically and numerical approximation becomes fundamental both for extracting quantitative solutions and for achieving a qualitative understanding of the behaviour of the solution.

In this paper we consider one-dimensional second-order scale-invariant nonlinear diffusion equations of the form

$$u_t = (uq)_x \qquad (a(t) < x < b(t)), \tag{1}$$

for a function $u(x,t)$, where q is of the form $\{p(u)_x\}^s$ with s an odd integer, and their radially symmetic counterparts, posed on finite moving domains. Typical boundary conditions for this problem consist of a Dirichlet condition on u and a flux condition on uv, where v is the boundary velocity. We assume here that $u = 0$ on the moving boundaries. In general the position of the boundary depends on the solution.

Many PDE problems that arise in practical applications possess scaling symmetries of the dependent and independent variables which are in some sense more fundamental than the equations themselves. In approximating such problems by numerical schemes it is desirable to construct algorithms that preserve these scaling properties, an objective beyond the reach of conventional numerical schemes based on fixed meshes in which the mesh depends on neither time nor the solution. The geometric integration of scale-invariant ordinary and partial differential equations (PDEs) was reviewed in Budd and Piggott in [11, 12] who considered the effectiveness of numerical methods in preserving the geometric structures of PDE problems, pointing to the need for moving-meshes (see also [13]).