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Stability and convergence of the higher projection method for the time-dependent viscoelastic flow problem[☆]

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Abstract

In this paper, the time discrete higher order projection method is proposed and analyzed for the time-dependent viscoelastic flow problem. Our numerical method is based on the time iterative discrete schemes. By the projection method, the considered problem is decoupled into two linear subproblems: One is for the velocity and the other is for the pressure. Unconditional stability of the numerical schemes is established. Convergence results for the velocity and pressure are also derived. Our main results of this paper are that the convergence analysis for the velocity is weakly second order and for the pressure is weakly first order. Finally, some numerical examples are provided to confirm the performances of the developed numerical algorithms.

Keywords: viscoelastic flow problem, higher order projection method, stability, error estimates
2000 MSC: 65N15, 65N30, 76D07

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1. Introduction

Let Ω be an open bounded domain in $\mathbb{R}^2$ with smooth boundary Γ. We consider the time-dependent viscoelastic flow problem with the initial-boundary conditions as follow:

$$\begin{cases} u_t - \nu\Delta u + (u\cdot\nabla)u + \nabla p - \int_0^t \rho e^{-\delta(t-s)}\Delta u ds = f, & (x,t)\in\Omega\times(0,T],\\ \text{div } u(x,t) = 0, & (x,t)\in\Omega\times(0,T],\\ u(x,0) = u_0(x), & (x,t)\in\Omega\times\{0\},\\ u(x,t) = 0, & (x,t)\in\Gamma\times(0,T], \end{cases} \tag{1.1}$$

where $\rho \geq 0$, $1/\delta$, $u = (u_1, u_2)^T$, p, f, and $u_0(x)$ represent the viscoelastic coefficient, the relaxation time, the velocity, the pressure, the prescribed external force, and the initial velocity, respectively, $T > 0$ is the finite time.

Problem (1.1) is used as a model in the viscoelastic flow problem [2, 3] because problem (1.1) is the generalization of the initial boundary value problem of the Navier-Stokes equation. For problem (1.1), many scholars have developed various kinds of numerical schemes to treat it. For example, we refer to [2, 3] for the existence and uniqueness of the solution of (1.1). For the numerical approximations, we can refer [18, 29, 30] for the penalty finite element method (FEM), [6, 11, 20] for the standard Galerkin FEM, and [19] for the linearized backward Euler discrete scheme. Based on the implicit/explicit scheme, the large time stepping viscosity-splitting method was studied at [31] while the stabilized characteristic FEM was investigated for problem (1.1) at [32]. Optimal error estimates have been established and verified by the numerical tests.

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